CMC3 Fall 2010 Attendance Report

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Contents

Chapter 1

Friday

This is a report [1] on my attendance at the California Mathematics Council, Community Colleges, 38th Annual Fall Conference. I've been attending these meetings off and on since 1990 and have always found them enlightening and invigorating. There's a video at the cmc.org site now of some of the original founders of CMC3 (including Pat Boyle, who gave me the idea for doing paper pop-ups for visualizing complex zeros in four dimensions) reminiscing about the founding of CMC3. They recount how at the seminal 1973 meeting, George Polya gave a talk about Galileo.

This year's opening talk was *A Piece of* π, given by John Martin of Santa Rosa Jr. College. Martin's speaking style is engaging, and most intriguing to me was the use of a "Question and Answer Sheet" (see addendum) contest, which was great fun. I scored 841, (21^2), but the top two scores were perfect, so no T-shirt for me.

Friday evening a publisher sponsored "game night," where you can join others attending the conference in Wii, chess, checkers, scrabble, and so on...oh, and Texas hold 'em.

I kibitzed at scrabble, wondered at Wii bowling for a while and then found my way to the poker game. A pair of kings in my pocket and some serious bluffers going to the limit brought me one big win with a full house (kings and aces) beating a flush. I thought or a while I might end up running the table, but I must have made some error in computing the payoff calculations on my patented Fano plane T.H.E. model, and I lost it all in the end. Win or lose, playing with this group was a blast!

Figure 1.1: Above is the lecture room at the Portola Hotel before the opening lecture. CMC3 President Barbara Illowski is next to Old Glory and to the left of her are a number of students. Pres. Elect Susanna Crawford is addressing the group.

Figure 1.2: I don't know everybody's name in this picture. Second from the right is past CMC3 president, Larry Green (Lake Tahoe CC)

[1] My second big attempt to use TeXnicenter software for LaTeX

Chapter 2

Saturday (Early)

The next morning was a bit fogged in with chance of tardiness for the 7:30 am estimation walk/run. I estimated I'd walk a mile in 31 minutes and 42 seconds. I dawdled, taking some nice pictures and pestering one returning group with, "Anyone have the time?" – As usual, one hapless contestant helpfully consulted her wristwatch (a no-no in this competition).

Figure 2.1: The group of walk/run estimation competitors, one of whom fell for the "have the time?" trap. Was this one of the groups who estimated their time exactly?

I returned about 3 minutes over time and got a free tee shirt for my efforts. I told Jay Lehman about what a troll I'd been to ask the others for the time and he joked that I was a typical bad student: late for the test and cheating. I started to protest that I had *caught* the cheaters, but thought better of it.

I retreated to my hotel room to finish reviewing the cmc3 2010 conference program and my plans for the day. I recount these as follows.

Figure 2.2: That's Jay Lehman (CMC3 Newsletter Editor, and textbook author) on the left and a friend (?) looking like opposing pawns on a big chessboard here.

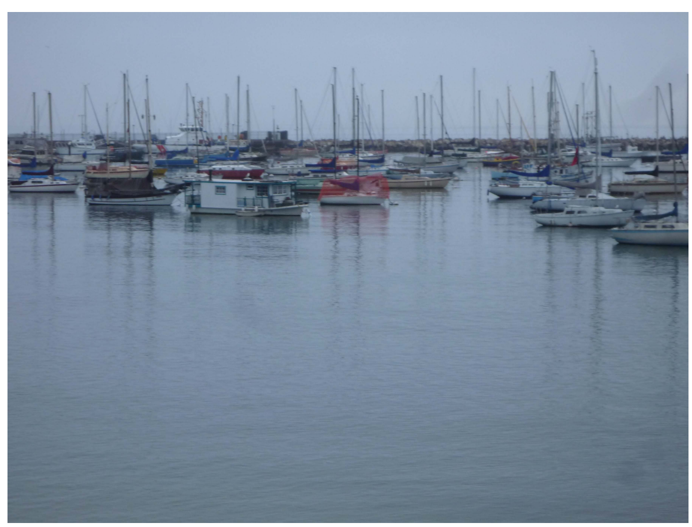


Figure 2.3: Sure is a lovely harbor just outside the door.

Chapter 3

Statewide Projects

Ian Walton of Mission College is a past president of ASCCC. He has an impressive record of service to the California Community Colleges! His presentation (available here) introduced me to some important statewide work, much of which I was unaware.

Figure 3.1: *The Statement on Competencies in Math logo.*

ICAS (Intersegmental Committee of Academic Senates) is a unified voice for higher education faculty in California whose achievements include the creation of a Statement on Competencies in Mathematics Expected of Entering College Students (SCMEECS) : the April, 2010 revision is available at their website. It amazes me that this document, officially endorsed by a coalition "representing the academic senates of the three segments of California's higher education system", was, until this meeting, unknown to me. I don't recall this relevant work was ever discussed at CalPASS meetings - I'll be sure to bring it up at the next meeting (January 18th?).

Given, then, that it is driven by teaching faculty representative of the three branches of higher ed in California, it is not surprising that SCMEECS has solid, trench-savvy pedagogical content and seems more concise and coherent than many similar documents I've studied. I hereby require everyone to increase their awareness of this documents and its significance to the curricula transitioning from k12 to higher education. Download the pdf to your Nook, or whatever, and start reading.

Walton was well aware of the disconnect between the fantasy that the SCMEECS espouses and the reality of public education in California, as it stands today. Still, the faculty that developed these expectations has done a fine job to spell out what the goals of a bachelor's degree are and the role of the community colleges vis-a-vis a host of official documents such as the UC a-g guidelines.

ICAS writes that the SCMEECS is offered,

> *...as their official recommendation on math preparation to the K-12 sector, to students and their parents, to teachers and administrators, and to public policy makers.*

ICAS makes the following basic recommendation in their introduction:

> *For proper preparation for baccalaureate level course work, all students should be enrolled in a mathematics course in every semester of high school. It is particularly important that students take mathematics in their senior year of high school, even if they have completed three years of college preparatory mathematics by the end of their*

junior year. Experience has shown that students who take a hiatus from the study of mathematics in high school are very often unprepared for courses of a quantitative nature in college and are unable to continue in these courses without remediation in mathematics.

Walton then turned to the ambitious CCCAssess project, a hot topic bringing many contentious programs together - something that is hoped will bear fruit in the long term. The goal is to create a "centrally delivered assessment system for California Community Colleges" and to develop assessment tools for (1) placement, (2) diagnostics and even (3) remediation in a wide range extending from 'whole numbers' to 'integration.' The questions of "linear" vs. "adaptive" tests and whether or not the assessment takes place in a single appearance, whether calculators are allowed, whether responses are multiple choice or written out, and what areas to test at what level - these are monumentally contraversial and difficult prescriptions to pin down in a static official policy, never mind the further difficulties in the development of tools to assess the policy, place students in the curriculum, diagnose their skills and prescribe remedial programs.

The discussion to this point in the group was lively and protracted. Wade Ellis and Barbara Illowski, pillars of CMC3 and a seminal members of the SCMEECS development group were in attendance and contributed much.

There wasn't much time left to discuss

- BRIC: The "Bridging Research, Information, and Culture initiative [that] aims to strengthen cultures of inquiry and evidence in the California community college system ..."
- The book, *Honored But Invisible*, Norton Grubb is an important read, especially for its perspective on the development of basic skills in the remedial curriculum (read a review of his book here.
- The importance of C-ID and SB1440.

Here's a little blurb from the *Honored but Invisible* web site that gets to what it's about:

This book examines the nature of teaching, and the institutional forces that shape it, in community colleges. These colleges include the most diverse students and the most varied subjects of any form of education in the U.S. Unfortunately, both they and the teaching within them are often invisible.

Here's a quote from the introduction:

[A] central conclusion of this book is that many community colleges as institutions pay little attention to teaching: They fail to use their institutional resources to enhance the quality of instruction, so that good teaching emerges only in isolated and idiosyncratic ways.

The author could well be referring directly to COD, where it has often seemed to me that institutional resources are prioritized in favor of hiring top administrators at the expense of fostering better teaching. COD is a teacher's college in that we teaching faculty are only paid to teach and research teaching. That's really good - it ought to be ballyhooed!

Chapter 4

A Tour of Free On-line Math Resources

Past CMC3 president, Larry Green, gave a virtual tour of many of the outstanding free tools for mathematics that are available online, such as videos, tutorial, animations, games and math creation applets. His presentation is available here.

The open textbooks project continues to grow, and this is promising for the hope of bringing down textbook prices.

Larry's done some great research in ferreting out instructional videos and even produced many of his own. I watched his video on finding an equation for the line tangent to a hyperbola, for instance, and found it quite accessible. There are elements of the explanation I would change, but I think this relates nicely to Grubb's ideas about creating a teaching community where our lectures are not so isolated from one another. Faculty in a department (or world-wide) can post lectures online and critique one another. Even I might learn some new tricks!

Green also has a nice collection of math applets. As he says: Enjoy!

Larry recommends Geogebra, Wolfram Alpha, CalcPlot3D, the Statistics Online Computational Resource (great new discovery for me! - Check out their experiments), and very nice Riemann Sums applet to boot.

Coming from Lake Tahoe, it's appropriate Larry is a big games afficianado (he teaches classes in gambling) and he advocates games in promoting active learning which is memorable and an effective alternate learning style for many students. He also warns against the pitfalls–for many students the frustration of difficulty in learning mathematical concepts can be compounded by losing at a game. His games site has a fantastic collection and I look forward to exploring it in more detail as time allows. While we're at it, here's a chance to become a millionaire!. Or just get goofy with simplifying linear expressions.

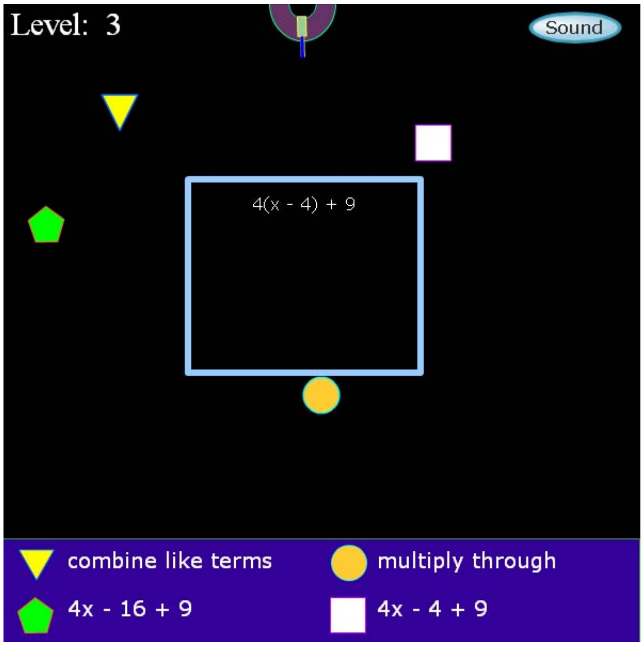


Figure 4.1: *A screenshot of The Simplifying Linear Expressions Game.*

Social networking issues vis-a-vis learning math via Facebook/Twitter were discussed. Well, this talk was really popular - the room became so crowded that I felt compelled to make room for others by going elsewhere – so later I followed the links in his posted presentation. The instructions for joining CCCConfer are easy enough and that looks like an excellent venue for community college folks to get together. Check it out–we can confer!

Along the same lines of social networking a la conference call is Google Wave. I have an account, but no one to talk to...very sad.

Finally, Larry gets to the category of "Meta Resources" such as the Multimedia Educational Resource for Learning and Online Teaching (MERLOT) which purports to be

Putting Educational Innovations Into Practice [by providing tools to] find peer reviewed online teaching and learning materials, share advice and expertise about education with expert colleagues and be recognized for your contributions to quality education.

MERLOT's mathematics portal offers a lot to explore and a venue for delivering material to students outside of the pay-per-view style profiteering of, say, Blackboard and the like.

Check out MERLOT's interactive lectures venue for, say, How Much Work is Required: Intuition vs. Mathematical Calculation. If you go to MERLOT's Learning Material, you can search for all sorts of things with much to find! Searching for "Larry Green" I got 16 hits–all items authored by Larry including links to games and videos like those discussed in his talk.

Former COD student and old friend, Ryan Ruff, who teaches at a community college in the Chesepeak Bay area, is intensely interested in the use of video games for education such as those advocated at The Cross-Cultural Rhetoric Blog. Ryan is quite a talented programmer. Check out his Para-Basketball!

Ryan and I got together and talked for a bit when was in town with his new bride, Ashleigh, for the new year's first week and we compared notes about teaching. He's been using this online tool he recommends: WIZIQ, where teachers can get free virtual (online) classroom technology.

We promised one another, sort of, to get back to the redevelopment of Algebra Arcade Reborn!

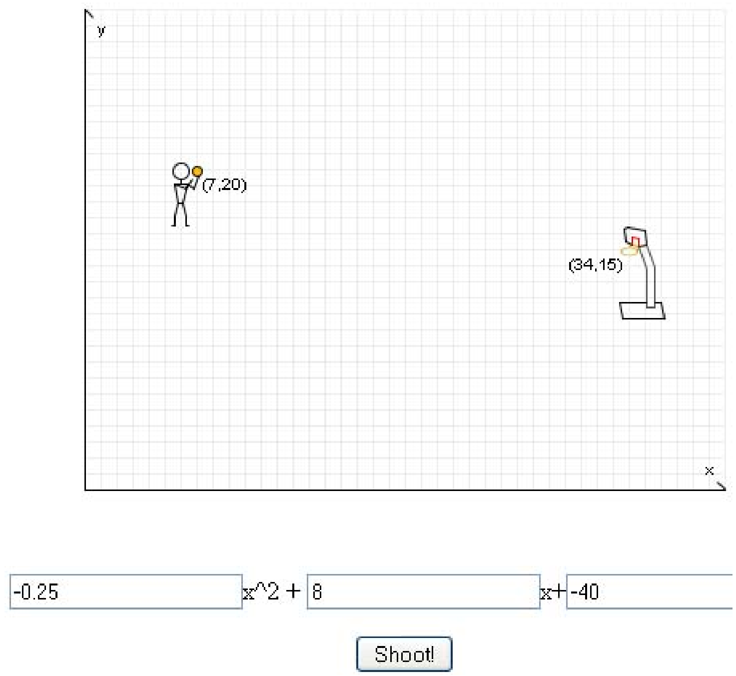


Figure 4.2: *A screenshot of Ryan Ruff's Para-Basketball game.*

Chapter 5

Two Morsels from Euler

The Keynote talk was a joy. William Dunham, author a book on my bookshelf: *Euler: Master of Us All* served up a couple of "side dishes" that suggest the genius of Euler in a way that makes you want to say, "Hey–I could have thought of that...if I were a genius, too."

Figure 5.1: *The Handmann portrait of Euler shows him wigless and in some kind of a Jimi Hendrix get up.*

Prof. Dunham started by giving a (very) brief overview of the vastness of the accomplishments of Euler, describing the Opera Omnia, an attempt to catalog the 866 and counting major works of Euler that has been ongoing since 1911, for instance, and then launched into the first (from 1742) of the two "morsels."

The first morsel came from a challenge by one of the Bernoullis to factor a quartic that had stumped everyone:

$$x^4 - 4x^3 + 2x^2 + 4x + 4$$

Euler showed that

$$x^4 - 4x^3 + 2x^2 + 4x + 4 =$$
$$(x^2 - (2 - \sqrt{4+2\sqrt{7}})x + (1 - \sqrt{4+2\sqrt{7}}))$$
$$*(x^2 - (2 + \sqrt{4+2\sqrt{7}})x + (1 + \sqrt{4+2\sqrt{7}})$$

And he showed how he did it.

Start with the goal of "depressing the quartic" (eliminating the cubic term) by substituting $x = y + 1$. This produces (after a short computation)

$$y^4 - 4y^2 + 7$$

-which is a big bonus because the first order term is now also gone so the expression is quadratic in y^2 –so the rest is easy!

The second morsel (1781) is a bit more involved. The challenge is to find four different whole numbers such that the sum of any two is a perfect square. Euler came up with the formidable foursome of 18530, 38114, 45986, and 65570. How'd he do it?

He started by assuming the numbers were

$$A < B < C < D$$

where

$$A + B = p^2 \quad B + D = s^2$$
$$A + C = q^2 \quad C + D = t^2$$

and to simplify matters:

$$B + C = A + D = r^2$$

where

$$p < q < r < s < t$$

This means that

$$2A + B + C = p^2 + q^2$$

and so

$$2A + r^2 = p^2 + q^2$$

whence, given the generators p, q, r, s and t we can produce

$$A = \frac{p^2+q^2-r^2}{2} > 0$$

so we need to require that

$$r^2 < p^2 + q^2$$

And from A, we get the others by

$$B = p^2 - A, C = q^2 - A, D = r^2 - A$$

Now, twice the square of r can be written

$$2r^2 = (A+D) + (B+C) = (A+B) + (C+D)$$
$$= p^2 + t^2$$

And. similarly,

$$2r^2 = (A+D) + (B+C) = (A+C) + (B+D)$$
$$= q^2 + s^2$$

These provoke the question:

When is $2r^2$ the sum of two squares?

The following equations suggest a pattern (at least to Leonhard):

$$5^2 = 3^2 + 4^2$$

and

$$2 \cdot 5^2 = 7^2 + 1^2$$

so, in general,

$$r^2 = x^2 + y^2 \Rightarrow r^2 = (x+y)^2 + (x-y)^2$$

The next question that occurred to Euler was when is it true that both r and r^2 a sum of two squares? For example,

$$5 = 2^2 + 1^2$$

and

$$5^2 = 4^2 + 3^2$$

Well, (obviously to LE) one could look at

$$r = x^2 + y^2 \Rightarrow r^2 = (x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$$

which means that if r is a sum of squares, then its square is also a sum of squares. Thus, thinks Leonhard, if r is a sum of squares and $r = m \cdot n$ where both m and n are sums of squares like say,

$$m = 2^2 + 1^2 = 5$$

and

$$n = 3^2 + 2^2 = 13$$

then

$$r = m \cdot n = 65 = 8^2 + 1^2 = 4^2 + 7^2$$

is a sum of squares in two different ways. Inspired by this fact, LE generalizes: if

$$m = a^2 + b^2$$

and

$$n = c^2 + d^2$$

then

$$r = m \cdot n = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$$
$$= (ac+bd)^2 + (ad-bc)^2$$
$$= (ac-bd)^2 + (ad+bc)^2$$

So the plan this suggests is to choose m and n each as the sum of two squares and then look at $r = m \cdot n$ and hope for the best.

A simple start is

$$m = 5 = 2^2 + 1^2, n = 13 = 3^2 + 2^2$$

and compute

$$r^2 = (mn)^2 = 65^2 = 4225$$

so that

$$2r^2 = 8450 = 23^2 + 89^2 = 47^2 + 79^2$$

But this is a dry hole since
$A = \frac{p^2+q^2-r^2}{2} = \frac{23^2+47^2-65^2}{2}$ is not positive.

So, try, try again. The next simplest guess might be

$$m = 5 = 2^2 + 1, n = 29 = 5^2 + 2^2$$

Then

$$mn = 145 \Rightarrow (mn)^2 = 21025$$

and

$$2r^2 = 42050 = 119^2 + 167^2 = 127^2 + 161^2$$

which means the generators are

$$p = 119, q = 127, r = 145, s = 161, t = 167$$

With these we get

$$A = \frac{p^2+q^2-r^2}{2} = \frac{119^2+127^2-145^2}{2} = \frac{9265}{2}$$

This is positive, but not an integer. But, hey–no problem! Just double all the generators, so that

$$p = 238, q = 254, r = 290, s = 322, t = 334$$

whence

$$A = \frac{238^2+254^2-290^2}{2} = 18530$$
$$B = p^2 - A = 38114$$
$$C = q^2 - A = 45986$$
$$D = r^2 - A = 65570$$

whence

$$A + B = 18530 + 38114 = 56644 = 238^2$$
$$A + C = 18530 + 45986 = 64516 = 254^2$$
$$A + D = 18530 + 65570 = 84100 = 290^2$$
$$B + C = 38114 + 45986 = 84100 = 290^2$$
$$B + D = 38114 + 65570 = 103684 = 322^2$$
$$C + D = 45986 + 65570 = 111556 = 334^2$$

Can you find another set?

Chapter 6

Mod, Map, and ASAP: Course Redesign at Pierce College

Katherine and Bruce Yoshiwara are familiar authors at COD, where a gulf has opened between those who advocate for using their Intermediate Algebra text and those who have used the "minimize completing the square" Sullivan text. Through their efforts to redesign the remedial curriculum, especially elementary and intermediate algebra, few leaders in the CCC math community have better epitomized what Norton Grubb in *Honored but Invisible* notes as the hallmark of an excellent teaching college, a college where

> *the faculty almost uniformly reported that their administrators are committed to teaching, and that there was no mistaking the institutional culture of innovation and experimentation (p. 311).*

Ms. Yoshiwara (with her husband, award winning math teacher, Bruce Yoshiwara, assisting) outlined what works and what still needs work in these three areas of course redesign that they've attempted:

1. ASAP (Algebra Success at Pierce) is a 14-unit learning community in 1 semester for total immersion in Math that consists of a 10 unit blended elementary/intermediate course, a 3 unit College Success course and a 1 unit Directed Study – study skills course.

2. MAP (Modeling with Algebra Project) Intermediate Algebra, features a discovery/directed learning approach with minimal lecturing and ISAs in the classroom.

3. MOD (Modularized) Algebra 1, a team taught, mastery approach with a complicated structure for advancing through 9 separate modules.

The slides for her presentation are available here.

The ASAP course is an intensive algebra immersion course where students study only elementary and intermediate algebra (and college success courses to make it a full load) for the semester. As Ms. Yoshiwara described it, 2/3 of the students succeeded in the ASAP course. This was significantly more that the roughly 50% of students who succeed in, especially when you consider that only 25% of traditional students successfully complete the two-course sequence. They also report that the ASAP students have the same success rate on their standardized exit exam.

If we were to try to replicate such a program at COD, an obvious question is what to call it? CODAS would be an acronym or College of the Desert Algebra Success, could be dubbed "happy CODAS," as in "happy endings."

Ms. Yoshiwara described the "MOD" design as not as successful as they might have wished. Many students simply failed to advance at the required mastery level and so the ranks of the lowest module swelled as students repeated it.

The "MAP" redesign is what the original Yoshiwara adopted at COD some 16 years ago has evolved into. This course features student discovery through directed learning activities. More is demanded from the student than in the traditional style texts. Y. recommended offering only one version of intermediate algebra, rather than offering side-by-side with what students perceive as "easier" intermediate algebra courses which demand less intellectual participation from the student. The scuttlebutt that "Prof. Easy" doesn't ask so much will out and the curriculum will become incoherent.

Here is a typical activity from Y. lexicon:

Activity: Slope and Linear Models

The taxi fare in three different cities is described below. In each city, you pay an initial charge when you get into the taxi, and then your fare is based on the distance you travel. Each city uses a different distance unit to compute the fare.

City	Initial Charge	Distance Unit	Charge per Unit
Boston	1.45	$\frac{1}{8}$ mile	0.30
Honolulu	2.25	$\frac{1}{4}$ mile	0.75
New York	2.50	$\frac{1}{5}$ mile	0.40

a. Compute the charge per mile in each city. (Do not include the initial charge.) In which city do taxis charge the highest mileage rate?

b. Write a linear model for the taxi fare in each city, using miles as the input variable. (**Hint:** What is the initial value of each model?)

c. In which city do taxis charge the lowest fare for a 5 mile ride?

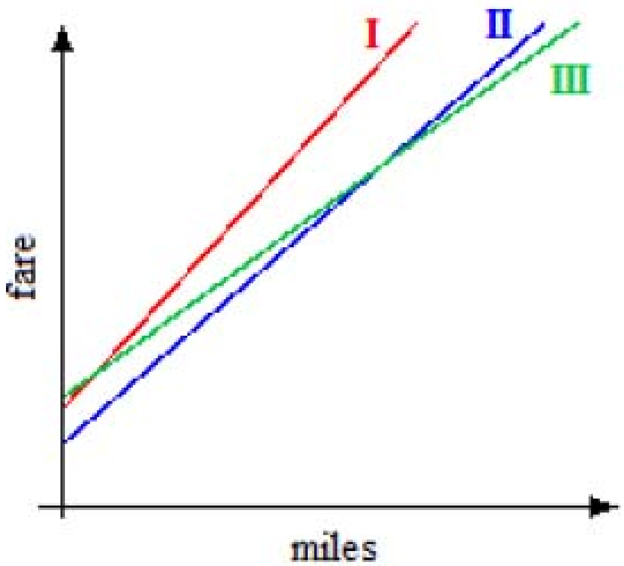
d. For what distance are the taxi fares in Boston and New York equal? (**Hint:** Use the appropriate models from part (b).)

e. Choose the correct graph fro each city.
Explain how you decided.

Boston:

Honolulu:

New York:



Y. asked the group which of these questions they thought would prove most difficult for the students at Pierce. It turned out the first question, seeminly simple in that it only require a division, say $0.30 \div \frac{1}{8}$, which is a basic skill alien to many high school graduates in California these days.

This redesign is accompanied by a robust set of WeBWork homework assignments. WeBWork describes itself as

> *an open source Perl-based system for delivering individualized homework problems over the web. [...] The major way in which WeBWorK differs from other web-based homework systems is in the way problems are written. The PG ("problem generation") language allows the inclusion of both Perl and LaTeX code, allowing problem authors to take advantage of the syntactic efficiency of Perl and the typographical flexibility of LaTeX (which is pretty much necessary for rendering mathematics expressions). The process of defining a problem is highly modular, with various pluggable display macros, answer evaluators, and graders. This allows for a high degree of freedom in defining how problems behave. A library of canned problems is provided, with over 12,000 problems contributed to the National Problem Library (or NPL) at last count. A large collection of macro files which simplify writing new problems is also available.*

In addition to Bruce Y's MAP problems, the U. of Rochester has a set of calculus homework to explore as a guest.

WeBWork is supported, in part, by a grant of the NSF and the MAA. I've requested to join the grant in developing problems for Elementary Algebra–we'll see what comes of it.

Chapter 7

Geodesics on Regular Polyhedra

U.C.Davis Prof. Dmitry Fuchs gave a lecture on *Geodesics on Regular Polyhedra*, confining his talk to the "seemingly simplest case" of geodesics on a polyhedral surface. "A point and an initial direction determine a whole geodesic," he writes and then poses the question, "Is it possible that a geodesic is close, that, after some time, starts repeating itself?" Almost nothing is known about the general case, but if the polyhedron is regular (a tetrahedron, cube, octahedron, icosahedron, or dodecahedron) there are exciting developments and beautiful pictures.

Figure 7.1: *Dmitry Fuchs and wife. A great humanitarian and a great mathematician!*

You can get a good flavor for Pr. Fuch's work through his paper Mathematical Omnibus: Thirty Lectures on Classic Mathematics. Lecture 20 is titled *Curvature and Polyhedra* and section 6 comes closest to what his lecture was about. There we find a discussion of closed geodesics on generic polyhedra. In figure 7.2 (from the Mathematical Omnibus) we see simple closed geodesics on a tetradedron and a cube: the section of the tetrahedron formed by a plane parallel to a pair of pairwise skew edges andthe suction of the cube formed by a plane perpendicular to a great diagonal.

Unfolding the triangular faces of the tetrahedron, T, onto a flat plane and labeling the vertices, A, B, C, D we

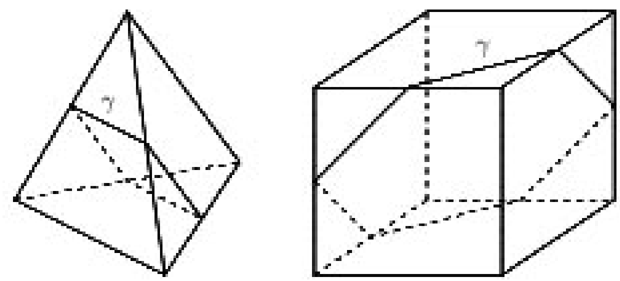


Figure 7.2: *Closed Geodesics on Polyhedra*

can tile the plane with replications of T. Prof. Fuchs, et al note (I paraphrase here) that

> Take the coordinate system in the plane with the origin at A and coordinate vectors $\vec{AB}, \vec{AC}$ as in the diagram. If $X = (\alpha, 0)$ is on $\vec{AB}$ as shown and $X' = (\alpha + 2p, 2q)$ is on another side labeled AB, p and q are relatively prime and $q\alpha \notin \mathbb{Z}$ then the map from the plane onto T takes XX' into a simple closed geodesic on T of length $\sqrt{p^2 + pq + q^2}$.

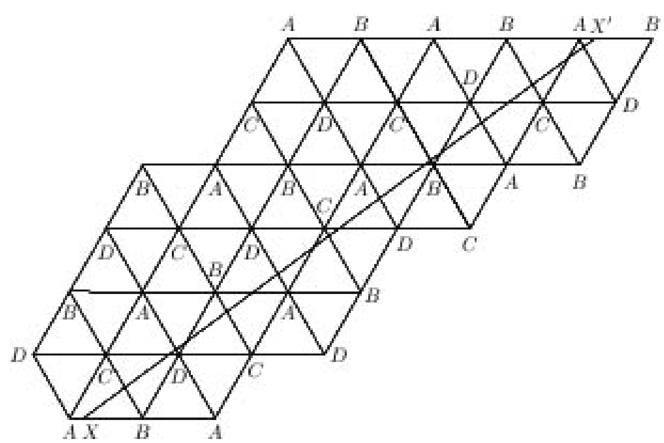


Figure 7.3: Triangular tiling with $p = 2$ and $q = 3$.

Further, this geodesic is non-intersecting, as are all non-intersecting geodesics on T and the geodesic cuts T into two pieces, as shown:

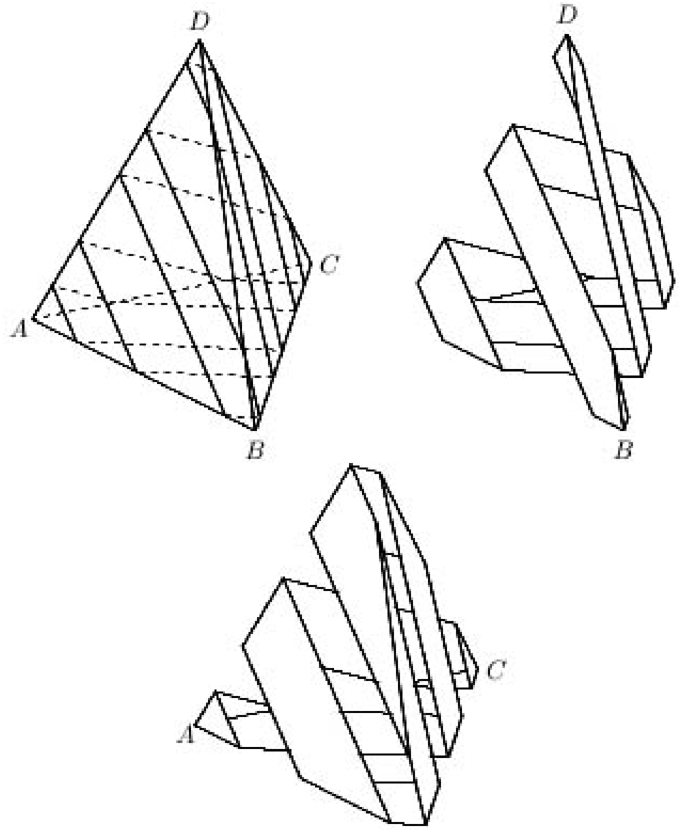


Figure 7.4: A closed geodesic on the tetrahedron.

In agreement with the Gauss-Bonet theorem, each piece contains two vertices.

In the *Omnibus*, Fuchs, et al, write that “A full description of closed geodesics on the regular octahedron is given in Exercise 20.9,” and, indeed, there are a whole bunch of tantalizing exercises at the end of this fabulous collection of lectures, but when you go to exercise 20.9 you find a reference to *D. Fuchs, E. Fuchs. Closed geodesics on regular polyhedra, Moscow Math. J., to appear.* Darn! A search of JSTOR for Dmitry Fuchs produces the article *More On Paper Folding*, which opens with this lovely statement of a theorem that the fold of a piece of paper is a straight line:

> *The model for a paper sheet is a piece of the plane; folding is an isometry of the part of the plane on one side of the fold to another, the fold being the curve of fixed points of this isometry. The statement is that this curve is straight, that is, has zero curvature.*

Which is followed by an elegant proof. Following this, Fuchs conducts a delightful investigation of curved folds...naturally!

I think it’s especially wonderful how the internets bring some of the joy of this discovery to those exiled to remote desert outposts: a window onto what you would otherwise be totally missing!

The Algebra and Topology Interaction sponsored a MSRI Conference in honor or Professor Dmitry Fuchs on the occasion of his 70th Anniversary with free public lectures such as *Flavors of “bicycle mathematics”* by S. Tabachnikov, of Penn State. His paper, On bicycle tracks geometry, [..] is available at the link. (As a frequent bicycle rider I’m happy to have this enrichment of the cycling experience!) Bicycle geometry defines

> *The **model of a bicycle** is a unit segment AB that can move in the plane so that it remains tangent to the trajectory of point A (the rear wheel is fixed on the bicycle frame.)*

Below are some types of these, in each the simpler, smoother, rear wheel track is in green. This may have been

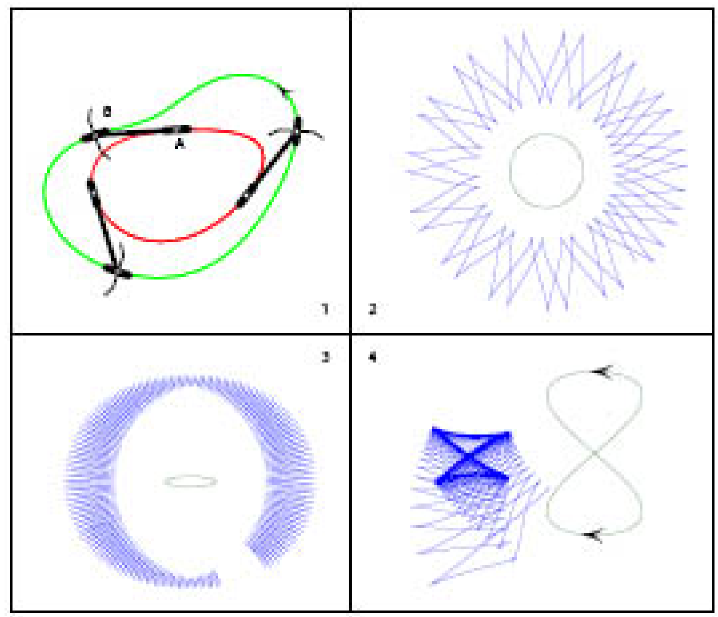


Figure 7.5: Examples 1 and 4 are hyperbolic; 2 and 3 are elliptic. The areas bounded by the two curves in 1 differ by πl^2.

inspired by *Which Way Did You Say That Bicycle Went?* by David L. FinnSource in Mathematics Magazine, Vol. 77, No. 5 (Dec., 2004).

Figure 7.6: *Which way did the bicycle go?*

Chapter 8

Appendix: The Pi Contest

The following pages contain the table built in an attempt to learn LaTeXby reproducing John Martin's handout for his talk, *A Piece of* π (see the *Friday* chapter) which I thought was a really effective learning tool - akin to the popular electronic responders used in classroom these days, but low tech pencil and paper. I suppose the same sort of quiz could be delivered with responders but...that might be half as much fun.

A Piece of π

Question and Answer Sheet

Directions: The following questions will be asked during the presentation. Answer each question after it has been posed and then decide how much you'd like to risk on your answer, anywhere from 0 up to 100 points. (You are given 100 points to start.) If your answer is correct, then add the amount risked to your total, otherwise subtract. Note that you may not risk more than 100 points on any question, and if your total becomes gegative, you are out of the game.

	RISK	BALANCE
1. Which one of the following formulas defines π? A. $\pi = Cd$ B. $\pi = \frac{d}{C}$ C. $\pi = \frac{C}{d}$ D. $\pi = \frac{C}{r}$ E. $\pi = 3.14$ (Where: d = diameter, C = circumference, and r = radius of a circle)		100
2. In which book will you find the following quotation? "He made the Sea of cast metal, circular in shape, measuring ten cubits from rim to rim and five cubits high. It took a line of thirty cubits to measure around it." A. The Bible B. The Qur'an C. Plimpton 322 D. Ahmes Papyrus E. Moscow Papyrus		
3. Which famous Greek scholar determined that $3\frac{10}{71} < \pi < 3\frac{1}{7}$? A. Archimedes B. Thales C. Pythogoras D. Martin E Eratosthenes		
4. In the fifth century A.D., a man and his son calculated π to 6 decimal places. In which country did they live? A. Greece B. China C. France D. Egypt E. Germany		
5. By what other name is Leonardo of Pisa known? A. DiCaprio B. Galileo C. da Vinci D. Lenny E. Finbonacci		
6. His expression for π was the first known use of an infinite product, whether connected with π or not. Who was he? A. Euler B. Gauss C. Jones Viète E. Wallis		
7. What other name has been used or π in Germany? A. die Euler Zahl B. die Pi Zahl C. el número bueno D. die Gesundheit E. die Ludolphasche Zahl		
8. In 1706 the Greek letter π was first used in print to represent the ratio of the circumference of a circle to its diameter. Who was the first to use it in this way? A Euler B. Gauss C. Jones C. Viète E. Wallis		
9. Which famous mathematician popularized the use of the symbol π when he switched from p to π in 1736? A. Euler B. Gauss C. Wallis D. Legendre E. Bernoulli		

10. What is the value of $e^{i\pi}$? A. $\sqrt{2}$ B. 1 C. 0 D. -1 E. $-\sqrt{2}$		
11. Which one of the following objects can be used to approximate the value of π by dropping it on a set of parallel line? A. a one dollar bill B. a penny C. a hula hoop D. a needle E. a haystack		
12. Which ancient geometrical problem did Ferdinand von Lendemann resolve in 1882 when he proved that π is transcendental? A. Trisection B. Duplication C. Squaring the Circle D. Rectification E. Fermat's Last Theorem		
13. In which state did Edwin J. Goodwin convince the legislature to consider (and nearly pass) his "new value" of π ? A. Texas B. Indiana C. New York D. Montana E. Georgia		
14. In 1873, William Shanks published his value of π to 707 decimal places. In which place did he make his first error? A 496 B. 526 C. 601 D. 666 E. 706		
15. What is the current record number of calculated digits of π ? A. 51.54 billion B. 206.16 billion C. 1.24 trillion D. 2.7 trillion E. 5 trillion		
16. What is the current world record number of memorized (and recited) digits of π ? A 12,345 B. 67,890 C. 83,431 D. 100,000 E. 314,159		