1. (12 points) Write without negative exponents and simplify.
   (a) $\left(\frac{3}{x}\right)^{-4}$ **Solution:** $\frac{x^4}{81}$
   (b) $\frac{2y^{-4}}{x^{-3}}$ **Solution:** $\frac{2x^3}{y^4}$
   (c) $\frac{3t^2(2t^{-3})^{-2}}{3t^{-3}}$. **Solution:** $\frac{t^{11}}{4}$

2. (12 points) Write each power in radical form.
   (a) $(12d)^{2/3}$ **Solution:** Any of these: $\sqrt[3]{(12d)^2} = \left(\sqrt[3]{12d}\right)^2 = \sqrt[3]{144d^2}$
   (b) $(9-4x^2)^{0.5}$ **Solution:** $\sqrt{9-4x^2}$

3. (10 points) Write each radical as a power with a fractional exponent.
   (a) $x\sqrt[3]{x}$ **Solution:** $x^{4/3}$
   (b) $\frac{1}{\sqrt[3]{x^2}}$ **Solution:** $x^{-2/3}$

4. (16 points) The heron population of Saltmarsh Refuge is estimated by conservationists at

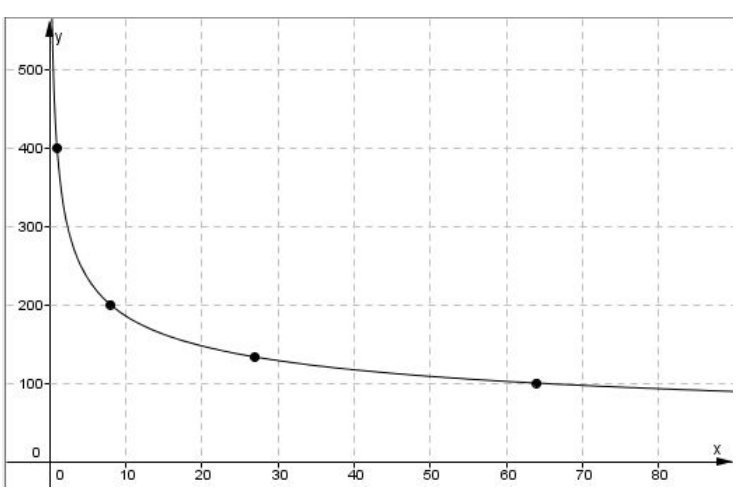
$$P(t) = 400t^{-1/3}$$

where t is the number of year since the refuge was established in 2000.
   (a) Complete the table

Solution:

t	0.001	1	8	27	64
$P(t)$	4000	400	200	133	100

   (b) Sketch a graph for the function for $0 \le t \le 64$



(c) Approximately how many heron were there in 2012?
From the graph, this number is approximately 170. We can evaluate the function to be more sure: $P(12) = 400(12)^{-1/3} = \frac{400}{\sqrt[3]{12}} \approx \frac{400}{2.894} \approx 175$

(d) In what year will there be only 10 heron left?
Solution: $P(t) = 10 \Leftrightarrow 400t^{-1/3} = 10 \Leftrightarrow \sqrt[3]{t} = 40 \Leftrightarrow t = 40^3 = 64000$ which corresponds to the year 68000. Hey, it's just around the corner.

5. (12 points) Write an equation for the circle

(a) with radius 2 and center $(1, 3)$.
$(x-1)^2 + (y-3)^2 = 4$

(b) centered at $(4, 5)$ and passing through $(1, 2)$.
The square of the radius is $(4-1)^2 + (5-2)^2 = 18$, so the equation is $(x-4)^2 + (y-5)^2 = 18$

(c) with diameter from $(-3, 2)$ to $(5, 4)$.
The center is at $\left(\frac{-3+5}{2}, \frac{2+4}{2}\right) = (1, 3)$ and the square of the radius is $(5-1)^2 + (4-3)^2 = 17$ so the equation is $(x-1)^2 + (y-3)^2 = 17$.

6. (12 points) Find the logarithm.

(a) $\log_2 64$ **Solution:** $\log_2 64 = 6 \Leftrightarrow 2^6 = 64$

(b) $\log_3 \frac{1}{27}$ **Solution:** ; $\log_3 \frac{1}{27} = -3 \Leftrightarrow 3^{-3} = \frac{1}{27}$

7. (12 points) Solve for the unknown value.

(a) $\log_2(1-2x) = 4$ **Solution:** $\log_2(1-2x) = 4 \Leftrightarrow (1-2x) = 2^4 \Leftrightarrow 2x = -15 \Leftrightarrow x = -\frac{15}{2}$

(b) $6 \cdot 10^{2.07x} = 216$ **Solution:** $6 \cdot 10^{2.07x} = 216 \Leftrightarrow 10^{2.07x} = 36 \Leftrightarrow 2.07x = \log(36) \Leftrightarrow x = \frac{\log(36)}{2.07} \approx 0.7943$

(c) $105 = \frac{1}{3}(10^{0.5x} - 1)$**Solution:** $105 = \frac{1}{3}(10^{0.5x} - 1) \Leftrightarrow (10^{0.5x} - 1) = 316 \Leftrightarrow 0.5x = \log(316) \Leftrightarrow x = 2(2.4997 \approx 4.9994$

8. (14 points) A obsessive saver puts 2¢ in a box on the first day of the month of January, 4¢ on the second day, 8¢ on the third day, and keeps doubling the number of cents she puts in the box for every day of the month thereafter.

(a) Make a table of the number cents she puts in the box each day for the first week.

Solution:

day	1	2	3	4	5	6	7
cents	2	4	8	16	32	64	128

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(b) Write a function which gives the number of cents she puts in the box on the nth day.
Solution: $A(n) = 2^n$

(c) How much money did she put in the box on the 15th day of the month.
Solution: $A(15) = 2^{15} = 32768$

(d) What if she had deposited the money in an account bearing 5% daily interest? How much money would she have at the end of the first week? You can write this as a sum, but you don't have to compute the sum.
Solution: The original 2¢ earns 6 day's interest, so it will be worth $2 \cdot (1.05)^6$¢ . Similarly, the 4¢ of day 2 will earn 5 day's interest and be worth $2 \cdot (1.05)^5$¢ , and so on, for a total of $2 \cdot (1.05)^6 + 4 \cdot (1.05)^5 + 8 \cdot (1.05)^4 + 16 \cdot (1.05)^3 + 32 \cdot (1.05)^2 + 64 \cdot (1.05) + 128 \approx 2.68 + 5.105 + 9.724 + 18.522 + 35.28 + 67.2 + 128 \approx 266.5$¢ .