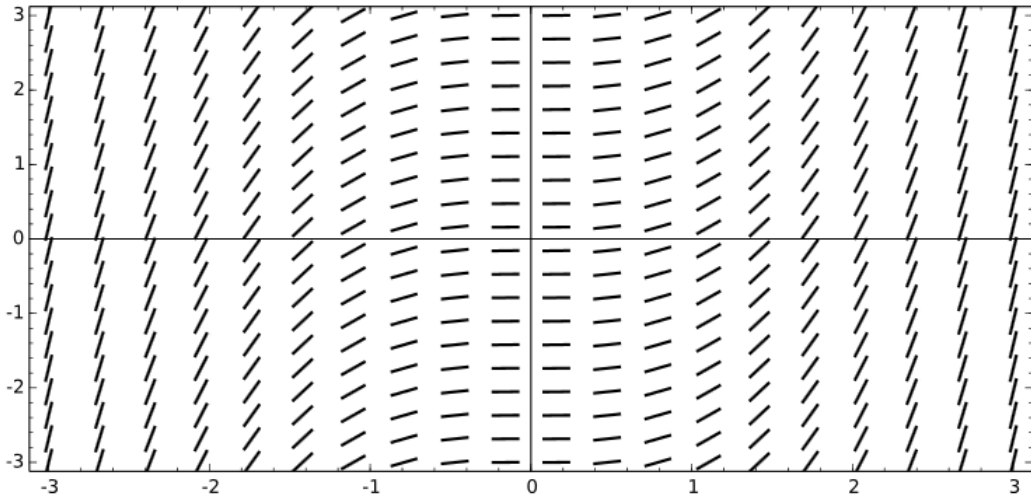


Write responses on separate paper.

- Suppose that $y(x)$ is a solution to the initial value problem $\frac{d}{dx}y(x) = f(x, y(x)), y(x_0) = y_0$. Express $y'(x_0)$ in terms of f, x_0 , and y_0 .
- Consider the directional field in the x, y -plane for $\frac{d}{dx}y = f(x, y)$ shown below.
 - What can you say about $f(x, y)$ based on the information in the direction field?
 - Approximate the initial value x_0 if $y_0 = -3$ and the point $(0, 0)$ is on the solution curve.



- Suppose that $y(x)$ is the solution to the IVP $x\frac{d}{dx}y = 2y$, where $y(1) = 3$. Find $y(2)$
- Find the general solution of the differential equation $\frac{d}{dx}y + \frac{2y}{x} = e^{x^3}$. Hint: use an integrating factor.
- Find the general solution of the differential equation $x\frac{dy}{dx} + y = 3xy$. Hint: use an integrating factor.
- Find $y(1)$ if $y(t)$ is the solution to the IVP $\frac{dy}{dt} + 2y = 2e^{-2t}$, with $y(0) = 2$.
- A tank originally contains 10 gal of water with 1/2 lb of salt in solution. Water containing a salt concentration of $\frac{1}{200}(10 - t)^2(1 + \sin(t))$ lbs per gallon flows into the tank at a rate of 1 gal/ min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/ min. The mixture is kept uniform by stirring. Let $Q(t)$ (in lb) be the amount of salt in the tank after time t (in min).
 - How long will it take for the tank to become empty?
 - Write the initial value problem for $Q(t)$ (before the tank is empty) and solve it.
- Solve the IVP: $(2xy + y^3)dx + (x^2 + 3xy^2)dy = 0$, where $y(1) = 2$
- Consider the system of ODEs, $\frac{d\vec{x}}{dt} = \begin{pmatrix} 4 & 2 \\ -1 & 6 \end{pmatrix} \vec{x}$
 - Find the general solution to the system.
 - Draw the phase portrait of the system, including the eigenlines. What is the origin called in this case? Is the origin stable, unstable or semi-stable?
- Use the method of undetermined coefficients to solve the initial value problem

$$y'' - 4y' + 13y = (4t - 4)\cos(3t) + (12t - 6)\sin(3t), y(0) = 1, y'(0) = 1$$

11. Find the solution of the given initial value problem

$$y'' + 4y = g(t), y(0) = 0, y'(0) = 0$$

where

$$g(t) = \begin{cases} 1, & t < \pi \\ \sin(t), & t \geq \pi \end{cases}$$

12. Show that the Laplace transform of $f(t) = t \sin(t)$ is $\frac{2s}{(s^2 + 1)^2}$ Hint: Use the relationship between the Laplace transform of a function and that of its derivative.

13. For the DE, $\frac{dy}{dx} = -\frac{y}{x} + 3x$

- Sketch the direction field for this DE, using (light or dotted) isoclines for the slopes $y' = -1$ and $y' = 0$.
- For the solution curve passing through the point $(1,2)$: if Eulers method with step-size $h=0.1$ was used to approximate $y(1.1)$, would the approximation come out too high or too low? Explain.
- Compute the Euler approximation to $y(1.1)$ using step-size $h = 0.1$.
- The functions $y_1 = x^2$ and $y_2 = x^2 + \frac{1}{x}$ are solutions to this DE. If $y = y(x)$ is the solution satisfying the IC $y(1) = 1.5$, show that $100 \leq y(10) \leq 100.1$. Do we need to include the equal signs in this inequality? Why or why not?
- Find the general solution the DE and verify the prediction of part (b).

14. Suppose that a population of variable size (in some suitable units) $P(t)$ follows the growth law

$$\frac{dP(t)}{dt} = P^3 - 4P^2 + 4P$$

Without solving the DE explicitly:

- Find all critical points and classify each according to its stability type using a phase-line diagram.
- Draw a rough sketch (on P -vs.- t axes) of the family of solutions. What happens to the population in the long-run if it starts out at size 1 unit?; at size 3 units?
- Explain why the rate information given by the DE was all we needed to get the answer to part (b).