SHORT ANSWERS TO MATH 223 Review Problems

1. (a) Decreasing
   (b) Increasing
2. A is a cylinder of radius 2 about the y-axis: $x^2 + z^2 = 4$
   B is two parallel planes: $x = \pm 3$
   C is a double cone: $x^2 + y^2 = z^2$
3. Horizontal circle of radius 2 centered at $(0, 0, 2)$
4. (a) a
   (b) b
5. (a) iii,viii; (b) iv; (c) vii; (d) ii; (e) v,vi; (f) i,ix
7. (b) (i) 1
   (ii) Increases
   (iii) Decreases
8. (a) Rose at 6 AM, directly overhead at 11:30 AM
9. False
10. (b) $f(x, y) = 2x - 3y - 2$
11. True
12. (a) $\frac{1}{\sqrt{6}}(\vec{i} - \vec{j} + 2\vec{k})$
    (b) $a = 8$
    (c) $\pm\frac{1}{\sqrt{3}}(-\vec{i} + \vec{j} + \vec{k})$
13. (c) Yes: $t \approx -0.5$
14. 458.23 lbs. at S 4.5° E
15. (b) $\vec{w} = -5\vec{i}$
    (c) $\vec{u} = 5\vec{i} + 2\vec{j}$
    (d) $\sqrt{29}\,\text{m/s}$
16. (a) True
    (b) False
    (c) False
    (d) False
    (e) False
17. (a) $\vec{n} = \vec{i} + 2\vec{j} + \vec{k}$
    (b) one possibility is $\vec{v} = \frac{2}{3}(\vec{i} + 2\vec{j} + \vec{k})$, $\vec{w} = \frac{1}{3}(\vec{i} - \vec{j} + \vec{k})$
    (c) $\frac{2}{3}\sqrt{6}$
18. (a) $\sqrt{6}$
    (b) $\frac{5}{6}\sqrt{6}$
19. (a) True
    (b) False
20. (a) $a = 3$
    (b) not possible
    (c) $10x + 7y + 3z = 20$
21. (a) $-12x + 4y + 3z = 5$
    (b) $\frac{13}{2}$
    (c) $\frac{13}{\sqrt{29}}$
22. $f_x(S) < f_y(Q) < 0 < f_x(P) < f_y(R)$
23. Positive
24. $\dfrac{2xy}{x^2y + 3}$
25. $\dfrac{4H + 3T + 10}{(5 - H)^4}$
26. (a) $\dfrac{\partial P}{\partial T} = \dfrac{R}{V}$, $\dfrac{\partial P}{\partial V} = -\dfrac{RT}{V^2}$
    (b) $-50\,\text{atm/m}^3$
27. $z = 3(x - 1)$
28. (c) $ds = \frac{1}{g}[2v\sin(2\alpha)\,dv + 2v^2\cos(2\alpha)\,d\alpha]$
    (d) α increases by 0.086
29. (a) Gets shallower at $4/\sqrt{17}$ feet per foot traveled
    (b) $\pm(3\vec{i} + \vec{j})$
30. $-\frac{10}{13}\sqrt{26}$
32. $2x + 4y + 3z = 36$
33. 37
34. (a) $\dfrac{\partial w}{\partial u} = 6\cos 1 + 6\sin 1$, $\dfrac{\partial w}{\partial v} = 6\cos 1 - 6\sin 1$
    (b) $-3e^{-1}$
35. (a) $\frac{3}{8}$
    (b) $49x^6y^6$
36. (a) $e^T \cdot \dfrac{e^P(P - 1) + T}{(T - e^P)^2}$
    (b) 2
    (c) $\pm\frac{1}{\sqrt{14}}(-\vec{i} - 3\vec{j} + 2\vec{k})$
    (d) $\dfrac{\partial P}{\partial V} = -\dfrac{nRT}{V^2}$
    (e) $\arccos\frac{1}{\sqrt{14}}$
37. (a) II,III,IV,VI
    (b) I
    (c) I,III,VI
    (d) VI
    (e) I,II,V
38. $\frac{36}{5}\sqrt{6}$
39. (b) $\displaystyle\int_2^{10}\int_1^{y/2} f(x, y)\,dx\,dy$
40. $V = 4(e^2 - 3)$

41. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{2-x^2-y^2}} dz\,dy\,dx$

42. (a) $\frac{1}{252}$
    (b) $\frac{1}{6}(\cos 5 - \cos 11)\sin 18$
    (c) $\frac{1}{2}(e^{16} - e^{9})$

43. $\displaystyle\int_0^{\pi}\int_0^{2} e^{-r^2} r\,dr\,d\theta = \tfrac{1}{2}\pi(1-e^{-4})$

44. (a) $\frac{2^{14}}{84}$
    (b) $-\frac{7}{3}$

45. (a) $\displaystyle\int_{\pi/2}^{3\pi/2}\int_1^4 \delta(r,\theta) r\,dr\,d\theta$
    (b) $\delta(r,\theta) = (4-r)(2+\cos\theta)$
    (c) $18(\pi-1)$

46. (a) $\frac{2}{3}$
    (b) $\frac{16}{3}$

47. (a) $\displaystyle\int_0^{2\pi}\int_0^{\pi/4}\int_0^{3} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$
    (b) $9\pi(2-\sqrt{2}\,)$

48. (a) $\displaystyle\int_0^{2\pi}\int_0^{2}\int_0^{3} (1+r) r\,dz\,dr\,d\theta$
    (b) 28π

49. π

50. (a) $x = 3\cos t, y = -3\sin t, \quad 0 \le t \le 2\pi$
    (b) $x = x_0 + 2\cos t, y = y_0, z = z_0 + 2\sin t, \quad 0 \le t \le 2\pi$

51. (a) $\vec{u} = 2\vec{i} + \vec{j} - 5\vec{k}$
    (b) one possibility is $\vec{u} = 2\vec{i} + \vec{j} + \vec{k}$
    (c) one possibility is $\vec{r}_0 = \vec{i} - \vec{k}$, $\vec{u} = \vec{i} + 8\vec{j} + 2\vec{k}$

52. $(1,5,3)$: $t = 0$, $(2,4,4)$: $t = 1$, $(0,6,2)$: $t = -1$

53. (a) $t = 4$
    (b) $\vec{r}(s) = \vec{i} + s\vec{j} + (10 - 2\pi - s)\vec{k}$

54. (a) 2 miles
    (b) 360 days
    (c) $\vec{v} = -\frac{\pi}{90}\vec{i}$, $\|\vec{v}\| = \frac{\pi}{90}$

55. (a) III; (b) V; (c) VI; (d) I; (e) II; (f) IV

56. (i) g; (ii) a; (iii) a; (iv) e; (v) e; (vi) d; (vii) f; (viii) b

58. (b) $\displaystyle\int_{C_1}\vec{F}\cdot d\vec{r} < \int_{C_2}\vec{F}\cdot d\vec{r} < \int_{C_3}\vec{F}\cdot d\vec{r}$

59. $\frac{21}{2}$

60. $10 - \cos 2$

61. (a) $f(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + C$
    (b) Not a gradient field

62. (b) C_1: $x = t, y = t, \quad 0 \le t \le 1$,
    C_2: $x = \cos\theta, y = \sin\theta, \quad 0 \le \theta \le 2\pi$
    (c) $\frac{5}{6}$
    (d) π

63. (a) A

64. (a) (i) Positive
    (ii) Negative
    (iii) Positive
    (iv) Positive
    (b) 20π

65. 45π

66. (a) $2 - 3x + 2xz$
    (b) 1

67. (a) Away from
    (b) $2(x^2+y^2)^a(1+a)$
    (c) Positive if $a > -1$, zero if $a = -1$, negative if $a < -1$

68. (c) $2xye^{x^2} + xe^{y} + xye^{y} + \cos xy$

69. (b) $10{,}000\pi$

70. (c) $2y\vec{i} - 2x\vec{j} + 2\vec{k}$

71. (a) (i) Zero, (ii) Zero, (iii) Zero, (iv) Could be
    (b) (i) Positive, (ii) Zero, (iii) Positive, (iv) Could not
    (c) (i) Zero, (ii) Positive, (iii) Zero, (iv) Could be

72. True

73. (b) -24π

74. False, False, False, True, True, False, False, True

75. First column: V, S, N, V, S
    Second column: S, N, V, N, S

76. i. 0
    ii. -7
    iii. -4
    iv. -3π