Name: ____________________

Recitation: ____________________

Multivariable Calculus

Math 215 Fall 2002

Final Exam

December 13, 2002

Please *do all your work in this booklet* and **show all the steps**.
Write your final answer *in the corresponding box*.
Calculators are not allowed.
One 3x5 note-card (both sides) is allowed.

Problem	Possible points	Score
1	15	
2	15	
3	10	
4	10	
5	20	
6	10	
7	10	
8	10	
Total	100	

Problem 1. *(15 pts.)*

(a – 5 pts.) *Find the rate of change of the function* $f(x, y) = 3x^2y + y^3$ *at the point* $(1, 2)$ *in the direction of* $\mathbf{v} = \langle -3, 4 \rangle$.

Answer: *(a)*

(b – 5 pts.) *Find the equation of the tangent plane to the surface given by the equation* $x\cos(z) - y^2 \sin(xz) = 2$ *at the point* $P(2, 1, 0)$.

Answer: *(b)*

(c – 5 pts.) *Find the distance from the point* $P(-3, 3, 4)$ *to the plane given by the equation* $6x - 2y + 3z = 9$.

Answer: *(c)*

| | 15 |

Problem 2. *(15 pts.)*

(a – 5 pts.) *A thin wire is in the shape of the helix parameterized by* $\mathbf{r}(t) = \langle 3\cos(t), 3\sin(t), t\rangle$, $0 \le t \le 4\pi$ *and has the linear density given by* $\delta(x, y, z) = y^2 + 1$. *Find the mass of the wire.*

Answer: *(a)*

(b – 5 pts.) *Find the mass of the cylindrical surface of radius* 5 *centered on the* x*-axis and bounded by the planes* $x = -3$ *and* $x = 3$, *if the surface density function of the cylinder is* $\delta(x, y, z) = |x|$.

Answer: *(b)*

(c – 5 pts.) *Find the mass of the region below the sphere of radius 3 centered at the origin, outside of the cylinder of radius 2, and above the xy-plane, if the density function is equal to the distance to the xy-plane.*

Answer: *(c)*

	15

Problem 3. *(10 pts.) Find the velocity and position vectors of a particle that has the acceleration* $\mathbf{a}(t) = 2t\mathbf{i} + 2\mathbf{j}$ *(m/sec*^{2}*) if its initial velocity is* $\mathbf{v}(0) = 3\mathbf{i} + 4\mathbf{k}$ *(m/sec), and its initial position is* $\mathbf{r}(0) = 4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$*. What is the distance the particle travels in the first 5 seconds?*

Answer:

Problem 4. *(10 pts.) Find the work done by the "inverse square" vector field* $\mathbf{F} = \dfrac{K\mathbf{r}}{r^3}$ *on a particle moving from* $(1, 0, 1)$ *to* $(1, 0, 0)$ *along the curve parameterized by* $x = \cos(2\pi t)$*,* $y = (t - t^2)$*, and* $z = (1 - t)$*,* $0 \leq t \leq 1$*. Here* $\mathbf{r} = \langle x, y, z \rangle$*,* $r = |\mathbf{r}|$*, and* K *is some proportionality constant.*

Answer:

	10
	10

Problem 5. *(20 pts.)* ***Green's Theorem, Stokes' Theorem and the Divergence Theorem.***

(a – 6 pts.) *Compare circulations of the vector field* $\mathbf{F} = \langle \arctan x + y^3, 2x - \sqrt[3]{y} \rangle$ *around the circles of radii 1 and 2 centered at the origin and oriented counterclockwise.*

Answer: *(a)*

(a – 7 pts.) *Compute the flux of the vector field* $\mathbf{F} = x^3\mathbf{i} + 2xz^2\mathbf{j} + 3y^2z\mathbf{k}$ *over the surface* S*, where* S *is the boundary of the solid bounded by the paraboloid* $z = 4 - x^2 - y^2$ *and the xy-plane.*

Answer: *(b)*

| | 20 |

(b – 7 pts.) Compute the integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$, $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + (x^2 + y^2)\mathbf{k}$, *where* C *is the positively oriented boundary curve of the part of the unit sphere* $x^2 + y^2 + z^2 = 1$ *in the first octant.*

Answer:

(c)

Problem 6. *(10 pts.) Find the maximum and the minimum values of the function* $f(x, y) = 4x^2 + xy + 4y^2$ *on the unit disk* $x^2 + y^2 \leq 1$ *and the points where these values are attained.*

Answer: *(Maximum)*

Answer: *(Minimum)*

| 10 |

Problem 7. *(10 pts.)*
True or False?

(1) *Any constant vector field* $\mathbf{F}$ *is a gradient vector field.* T F

(2) *If a line integral of a vector field* $\mathbf{F}$ *along a unit circle* $x^2 + y^2 = 1$ *is zero, then* $\mathbf{F}$ *is a gradient vector field.* T F

(3) *If* C_1 *and* C_2 *are two oriented curves,* $\mathbf{F}$ *is a vector field, and the length of* C_1 *is greater than the length of* C_2*, then* $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} > \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$. T F

(4) *The integral* $\int_C \mathbf{F} \cdot d\mathbf{r}$ *does not depend on the parameterization of* C. T F

(5) *If* $\operatorname{div}(\mathbf{F}) = 0$ *and* $\operatorname{curl}(\mathbf{F}) = \mathbf{0}$*, then* $\mathbf{F} = \mathbf{0}$. T F

(6) *If* $\operatorname{curl}(\mathbf{F}) = \mathbf{0}$*, then* $\mathbf{F}$ *is a conservative vector field.* T F

(7) *If* $\mathbf{F}$ *is a gradient vector field, then* $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$ *for any smooth oriented surface* S. T F

(8) *There exists a vector field* $\mathbf{F}$ *such that* $\operatorname{curl}(\mathbf{F}) = x\mathbf{i}$. T F

(9) *If* S_1 *and* S_2 *are two oriented surfaces bounded by the same positively oriented curve* C *and* $\mathbf{F}$ *is a smooth vector field, then the flux of* $\operatorname{curl}(\mathbf{F})$ *through* S_1 *and* S_2 *is the same.* T F

(10) *If* S *is a unit sphere centered at the origin and* $\mathbf{F}$ *is a vector field satisfying* $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$*, then* $\operatorname{div}(\mathbf{F}) = 0$ *at all points inside* S. T F

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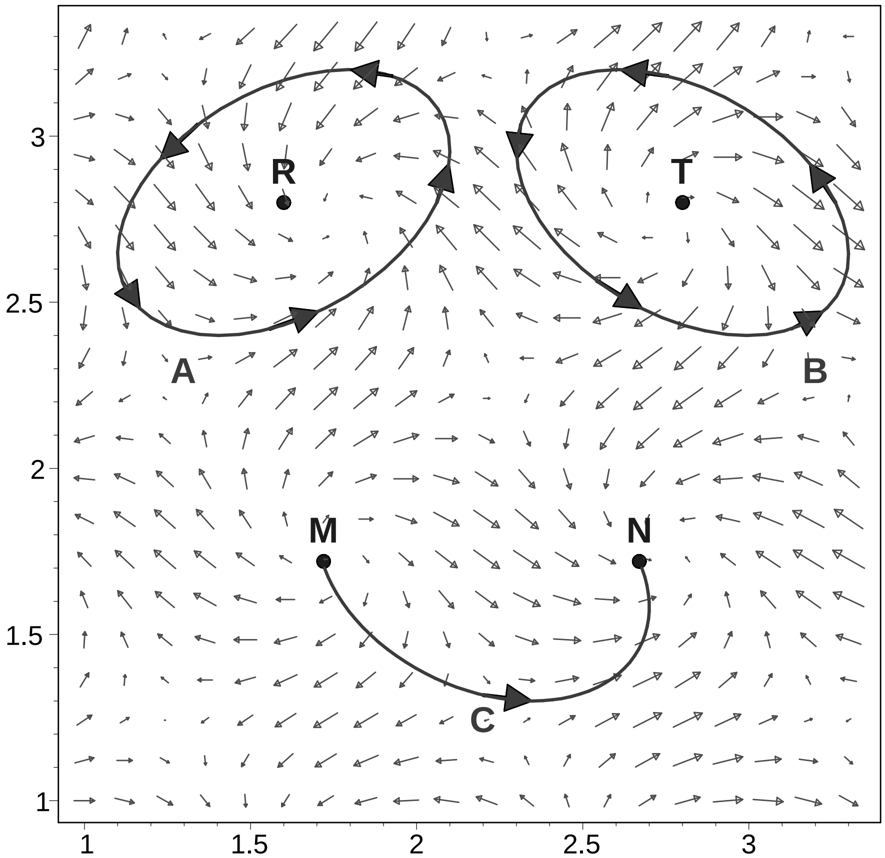
Problem 8. *(10 pts.)* *The graph below is a plot of some planar vector field* $\mathbf{F} = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$, *oriented curves* A, B, C, *and points* R, T, M, N. *Use this information to answer the following questions.*



(a) *Is the circulation of* $\mathbf{F}$ *around the curve* A *positive, negative, or zero?* ________________

(b) *Is* $\oint_B \mathbf{F} \cdot d\mathbf{r}$ *positive, negative, or zero?* ________________

(c) *Is the work done by* $\mathbf{F}$ *in moving a particle from* M *to* N *along* C *positive, negative, or zero?* ________________

(d) *Is the value of* $Q_x - P_y$ *at* R *positive, negative, or zero?* ________________

(e) *Does the curl of* $\mathbf{F}$ *at* T *point in the direction of the positive or negative* z*-axis or is it zero?* ________________

(f) *Is the divergence of* $\mathbf{F}$ *at* M *positive, negative, or zero?* ________________

(g) *Is the divergence of* $\mathbf{F}$ *at* N *positive, negative, or zero?* ________________

(h) *What would a small paddle placed at* M *do?* ________________

(i) *What would a small paddle placed at* N *do?* ________________

(j) *Is* $\mathbf{F}$ *a conservative vector field?* ________________

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