MA 174: Multivariable Calculus

EXAM I (practice)

NAME ______________________ INSTRUCTOR ______________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) ___________	**9. (5 pts)** ___________
2. (5 pts) ___________	**9. (5 pts)** ___________
3. (5 pts) ___________	**9. (5 pts)** ___________
4. (5 pts) ___________	**9. (5 pts)** ___________
5. (5 pts) ___________	**9. (5 pts)** ___________
6. (5 pts) ___________	**9. (5 pts)** ___________

Total Points: ___________

1. **Find the normal vector to the plane** $3x + 2y + 6z = 6$

   A. **(0, 0, 1)**

   B. $\boxed{\textbf{(-3, -2, -6)}}$

   C. **(1, 1, 1)**

   D. **(0, 0, 1)**

   E. **(1/3, 1/2, 1/6)**

2. **Find distance from point** $(1, 1, 3)$ **to the plane** $3x + 2y + 6z = 6$

   A. $\boxed{\textbf{17/7}}$

   B. **3**

   C. **12/5**

   D. **0**

   E. **11/7**

3. **The surface defined by** $y^2 - x^2 = z$ **is a**

   A. $\boxed{\textbf{hyperbolic paraboloid}}$

   B. **elliptical cone**

   C. **elliptical paraboloid**

   D. **ellipsoid**

   E. **hyperboloid**

4. **Find the speed of the particle with position function $\vec{r}(t) = e^{3t}\,\mathbf{i} + e^{-3t}\,\mathbf{j} + te^{3t}\,\mathbf{k}$ when $t = 0$.**

A. $\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$

B. 1

C. $\sqrt{2}$

D. $\sqrt{17}$

E. $\boxed{\sqrt{19}}$

5. **The plane S passes through the point $P(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, and $z = 2 - t$. Which of the following vectors is normal to S?**

A. $\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$

B. $\boxed{\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}}$

C. $\mathbf{i} + \mathbf{k}$

D. $\mathbf{i} - 2\,\mathbf{j}$

E. $\mathbf{i} + 2\,\mathbf{j}$

6. **Which of the following statements is true for all three-dimensional vectors $\vec{a}, \vec{b}$, and $\vec{c}$, if θ is the angle between $\vec{a}$ and $\vec{b}$?**

(i) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

(ii) $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{b} \times \vec{c}) \cdot \vec{a}$

(iii) $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot |\cos\theta|$

(iv) $(\vec{a} \times \vec{b}) \cdot a = 0$

A. **All are true**

B. **(i) and (ii) only**

C. **(i), (ii), and (iv) only**

D. **(iii) and (iv) only**

E. $\boxed{\textbf{(ii) and (iv) only}}$

7. The level surface of $f(x, y, z) = x^2 + y^2 - z^2$ corresponding to $f(x, y, z) \equiv 1$ intersects the xy-plane in a

A. $\boxed{\textbf{circle}}$

B. parabola

C. ellipse

D. hyperbola

E. line

8. If $L = \lim_{(x,y,z)\to(0,0,0)} \dfrac{x + 2y - 3z}{\sqrt{x^2 + y^2 + z^2}}$**, then**

A. $L = 1$

B. $L = -2$

C. $L = -3$

D. $L = 0$

E. $\boxed{\textbf{the limit does not exist}}$

9. If $f(x, y) = \ln(x^2 + 2y^2)$**, then the partial derivative** f_{xy} **equals**

A. $\dfrac{4xy}{(x^2 + 2y^2)^2}$

B. $\dfrac{4(x^2 - y^2)}{(x^2 + 2y^2)^2}$

C. $\boxed{\dfrac{-8xy}{(x^2 + 2y^2)^2}}$

D. $\dfrac{-4y}{(x^2 + 2y^2)^2}$

E. $\dfrac{-2x}{(x^2 + 2y^2)^2}$

10. A particle starts at the origin with initial velocity $\vec{i}+\vec{j}-\vec{k}$. Its acceleration is $\vec{a}(t)=t\vec{i}+\vec{j}+t\vec{k}$. Find its position at $t=1$.

A. $\frac{1}{6}\ \vec{i}+\frac{1}{2}\ \vec{j}+\frac{1}{3}\ \vec{k}$

B. $\frac{7}{6}\ \vec{i}+\frac{1}{2}\ \vec{j}-\frac{5}{6}\ \vec{k}$

C. $\vec{i}+\vec{j}+\vec{k}$

D. $\boxed{\frac{7}{6}\ \vec{i}+\frac{3}{2}\ \vec{j}-\frac{5}{6}\ \vec{k}}$

E. $\vec{i}+2\vec{j}-\vec{k}$

11. Find the arc length of the curve defined by $\vec{r}(t)=(t,\frac{\sqrt{6}}{2}t^2,t^3), -1\leq t\leq 1$.

A. 5

B. $\boxed{4}$

C. 3

D. 2

E. 6

12. Find the equation of the plane that contains the points $(1,2,1)$, $(2,-1,0)$ and $(3,3,1)$.

A. $-x-2y+9z=4$

B. $x+2y+7z=12$

C. $\boxed{x-2y+7z=4}$

D. $x+2y+z=6$

E. $-x+2y+9z=12$

13. Find parametric equations for the tangent line to the curve

$$\vec{r}(t) = (t^2 + 3t + 2, e^t \cos t, \ln(t+1))$$

at $t = 0$**.**

A. $\boxed{x = 2 + 3t \quad y = 1 + t \quad z = t}$

B. $x = 2t + 3, \ y = e^t(\cos t - \sin t), \ z = \dfrac{1}{t+1}$

C. $x = 3 + 2t \quad y = 1 + t \quad z = 1$

D. $x = 3t \quad y = 2t \quad z = 1 + t$

E. $x = 2 - t \quad y = 1 + t \quad z = 3 - 3t$

14. Let C **be the intersection of** $x^2 + y^2 = 16$ **and** $x + y + z = 5$**. Find the curvature at** $(0, 4, 1)$**.**

Answer: $\frac{1}{8}\sqrt{\frac{3}{2}}$

15. Find the equation for the surface consisting of all points P **for which the distance to the** x**–axis is twice the distance from** P **to the** yz**–plane. Identify the surface.**

Answer: $y^2 + z^2 = 4x^2$**, elliptical cone**

16. Find an equation of the plane that passes through the point $P(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

Answer: $x - 2y + 4z = -1$

17. (a) Find the point of intersection of the lines $x = 2t + 1$, $y = 3t + 2$, $z = 4t + 3$ and $x = s + 2$, $y = 2s + 4$, $z = -4s - 1$.

(b) Find the plane determined by these lines.

Answer: (a) $x = (1, 2, 3)$, **(b)** $20x - 12y - z = -7$

18. A spring gun at ground level fires a golf ball at an angle of 45 degrees. The ball lands 10m away.

(a) What was the ball's initial speed?

(b) For the same initial speed, find the two firing angles that make the range 6m.

Recall that the Ideal Projectile Motion Equation is

$$\mathbf{r} = (v_0 cos\alpha)t\mathbf{i} + \left((v_0 sin\alpha)t - \frac{1}{2}gt^2\right)\mathbf{j}.$$

Answer: (a) $v_0 = \sqrt{10g}$,
Answer: (b) $\alpha = \frac{1}{2}$**arcsin**$(\frac{3}{5}), \alpha = \pi - \frac{1}{2}$**arcsin**$(\frac{3}{5})$

19. Find the unit tangent vector T, the principle unit normal vector N and the unit binormal vector B of $\mathbf{r(t)} = (\mathbf{3}\sin(\mathbf{t}))\mathbf{i} + (\mathbf{3}\cos(\mathbf{t})\mathbf{j} + \mathbf{4tk}$ at any t.

Recall: $\mathbf{N} = \frac{\frac{d\mathbf{T}}{dt}}{|\frac{d\mathbf{T}}{dt}|}$ and $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.

Answer: $\mathbf{T} = (\frac{\mathbf{3}}{\mathbf{5}}\cos(\mathbf{t}), -\frac{\mathbf{3}}{\mathbf{5}}\sin(\mathbf{t}), \frac{\mathbf{4}}{\mathbf{5}})$
Answer: $\mathbf{N} = (-\sin(\mathbf{t}), -\cos(\mathbf{t}), \mathbf{0})$
Answer: $\mathbf{B} = (\frac{\mathbf{4}}{\mathbf{5}}\cos(\mathbf{t}), -\frac{\mathbf{4}}{\mathbf{5}}\sin(\mathbf{t}), -\frac{\mathbf{3}}{\mathbf{5}})$

20. Calculate the tangential and normal components of the acceleration for $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + \frac{1}{3}t^3\vec{k}$.

Recall $a = a_T\mathbf{T} + a_N\mathbf{N}$ and $a_T = \frac{d}{dt}|v|$

Answer: $a_T = 2t, a_N = 2$

21. Find $\frac{\partial z}{\partial y}$ at $(1, \ln 2, \ln 3)$ if $z(x, y)$ is defined by the equation

$$xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0$$

Answer: $-\frac{5}{3\ln 2}$