Some Math 2A Problems for Chapter 12 Test Study – Spring '11

1. True or False or Conditional. Explain your answer.
   a. Any two vectors determine a plane.
   b. Any two lines determine a plane.
   c. For a given point and a given plane there is a unique line through that point and perpendicular to the plane.
   d. The intersection of any two planes is a line.
   e. Any line in the xy plane will intersect a line in the xz plane.

2. Find two vectors parallel to the plane $z = 3 - 2x + y$ but not parallel to each other and compute the cross product of these vectors.

3. Consider the point (1,1,3) and the plane $3x + 2y + 6z = 6$
   a. Find the point where the plane intersects the *z*-axis and

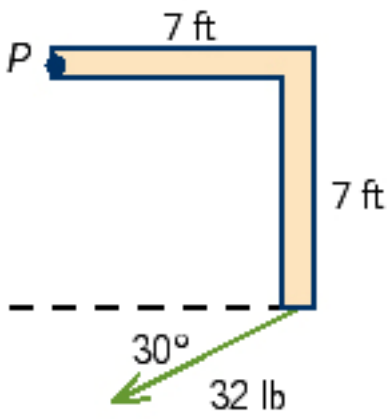
4. Show that the length of a vector is zero if and only if all its components are zero.

5. Identify the surface whose equation is given as one of the following: a hyperbolic paraboloid, an elliptical cone, an elliptical paraboloid, an ellipsoid, a hyperboloid of one sheet or a hyperboloid of two sheets.
   a. $x^2 - 2y^2 + 3z^2 = 6$
   b. $x^2 - 2y^2 - 3z^2 = 6$
   c. $x^2 + 2y^2 + 3z^2 = 6$
   d. $x^2 - 2y^2 + 3z^2 = 0$
   e. $x^2 + 2y^2 + z = 4$

6. The plane *S* passes through the point *P*(1, 2, 3) and contains the line $x = 3t$, $y = 1 + t$, and $z = 2 - t$. Find a vector normal to *S*.

7. Which of the following statements is true for all three-dimensional vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, if θ is the angle between $\vec{a}$ and $\vec{b}$? Note that none or all could be true.
   a. $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$
   b. $\vec{a} \bullet \left(\vec{b} \times \vec{c} \right) = \left(\vec{b} \times \vec{c} \right) \bullet \vec{a}$
   c. $\left| \vec{a} \times \vec{b} \right| = \left| \vec{b} \right| \cdot \left| \vec{a} \right| \cdot \left| \cos\theta \right|$
   d. $\left| \vec{a} \times \vec{b} \right| \bullet \vec{a} = 0$

8. Find the torque at P if a 32 pound force is applied to the rigid body shown in the diagram at right. Note that this is a planar diagram.



9. Describe the cross section that the given plane makes with the surface $z = x^2 - 2y^2 - 4xy$
   a. $x = 3$
   b. $y = x$
   c. $y = \left(1 + \frac{\sqrt{6}}{2}\right)x$

10. The Parallelogram Law states that $\left|\vec{a}+\vec{b}\right|^2 + \left|\vec{a}-\vec{b}\right|^2 = 2\left|\vec{a}\right|^2 + 2\left|\vec{b}\right|^2$
    a. Give a geometric interpretation of this law.
    b. Prove the law. The triangle inequality and/or Cauchy Schwarz inequality may be useful.

11. Find the equation of the plane that contains the points (1, 2, 1), (2, –1, 0) and (3, 3, 1).

12. Find the distance between the planes $z = 1 - x - 3y$ and $x + 3y + z = 5$

13. Find an equation for the line where the plane $x - 2y + z = 0$ intersects the plane $2x - y + 2z = 3$.

14. Parameterize the line segment from (6,4,1) to (3,2,5) as $0 \le t \le 1$.

15. Use vectors to prove that diagonals of a rhombus are perpendicular.

16. Find the area of a parallelogram formed by vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ if $P(1,2,3)$, $Q(5,4,2)$ and $R(7,2,5)$.

17. Show that for all $a, b \in \mathbb{R}$, if $x > 0$ and $y > 0$, $\frac{(a+b)^2}{x+y} \le \frac{a^2}{x} + \frac{b^2}{y}$.

18. Write the equation in standard form: $x - y^2 + 8y + 4z^2 + 4z = 115$ and describe the surface.

19. Suppose that Jack and Jill pull on a ropes attached to an object. Jack pulls with a force of 450 N and Jill pulls with a force of 300 N. The angle between the ropes is 30°. With what direction and force should a third person pull so as to keep the object from moving? Draw a diagram.

20. What force (in Newtons) must be applied to the end of a lever of length 20 cm in the direction $\langle 0, 3, 4 \rangle$ to produce a torque of magnitude 10 Nm?

21. How can the triple product be used to determine whether or not three different vectors are coplanar? Give and example.

22. Find the cross product of $\left\langle 1, \cos\frac{\pi}{3}, \sin\frac{\pi}{6} \right\rangle \times \left\langle 1, \cos\frac{\pi}{6}, \sin\frac{\pi}{2} \right\rangle$ and simplify.

23. Find the cross product of $\langle 1, \cos\theta, \sin\theta \rangle \times \langle 1, \sin\theta, \cos\theta \rangle$ and simplify.

24. Find the length of the projection of a vector from any point P on the plane $x + 2y - 3z = 6$ to $Q(1,3,2)$ onto a vector normal to the plane.

25. Think of two quadric surfaces whose intersection is the ellipse $x^2 + 3y^2 = 1$ in the plane $z = 1$.