# The Ant and the Sugar Bowl

Write your solution to the following problem in expository form. That is, don't just write a bunch of unexplained equation and formulas – explain what you are doing as you do it. You may work with others in the class, but do not copy other people's work. This counts as half a test score.

Problem:

An ant at the bottom of an almost empty sugar bowl eats the last few remaining grains. It is now too bloated to climb at a vertical angle as ants usually can; the steepest it can climb is at an angle $\alpha$ to the horizontal with a tan $\alpha = 1$. The sugar bowl is shaped like a paraboloid, $z = x^2 + y^2$, $0 \leq z \leq 4$ were the coordinates are in centimeters.

a) Find the path the ant takes to get to the top of the sugar bowl, assuming it climbs as steeply as possible. Use polar coordinates $(r, \theta)$ in the $xy$-plane and think of the ant's path as parameterized by $r$ and $\theta$; then find a relation between the differentials $d\theta$ and $dr$, and integrate this relation to get $\theta(r)$.

b) What is the length of the ant's path from the bottom to the rim? To answer this, first discover a formula for arc length involving $dz$, $dr$, and $d\theta$ in three dimensions.

c) Draw a graph of the sugar bowl and the path the ants takes to get out. Hint: you may want to start with the projection $\theta(r)$ of the path in the $r\theta$-plane.