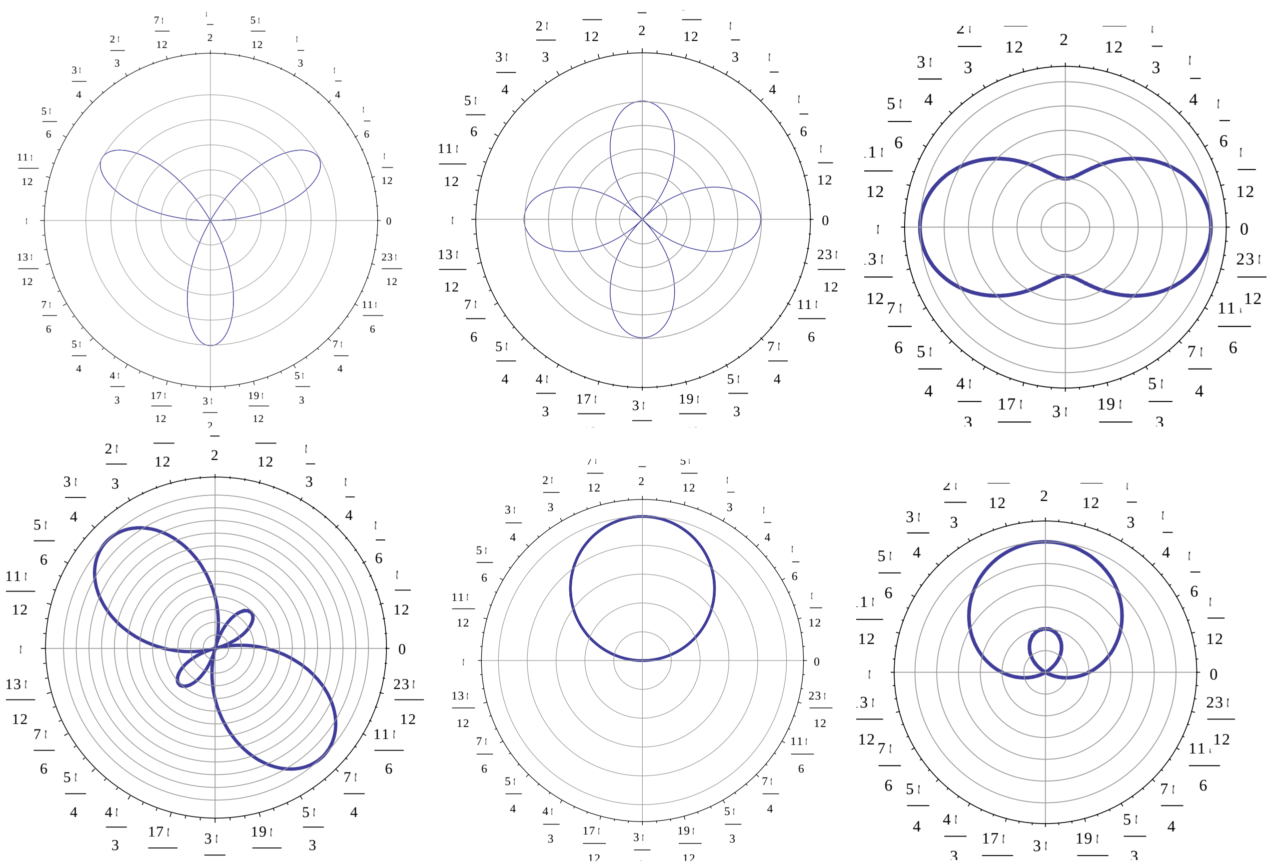
Precalculus – Math 12 – fall '09 – Test 1.   Name____________________________

Write all responses on separate paper. Show all work for credit. No calculator is needed for this test.

1. Match the graph with one of the formulas.
   a. $r = \sin\theta$
   b. $r = 1 + 2\sin\theta$
   c. $r = \sin 3\theta$
   d. $r = \cos 2\theta$
   e. $r = 2 + \cos 2\theta$
   f. $r = 1 - 2\sin 2\theta$



2. Convert the point whose polar coordinates are given to rectangular coordinates.
   a. $(r,\theta) = \left(2, \frac{5\pi}{4}\right)$ b. $(r,\theta) = \left(4, \frac{11\pi}{6}\right)$

3. Find two polar coordinate representations for the rectangular coordinate point, $(x, y) = \left(2, -\sqrt{3}\right)$ one with $r > 0$ and one with $r < 0$, and both with $0 \le \theta < 2\pi$.

4. Consider the polar equation $r = cos\theta + sin\theta$.
   a. Find an equivalent rectangular form for this equation and write it in standard form.

b. Tabulate values of r corresponding to $\theta = 0, \pm\frac{\pi}{6}, \pm\frac{\pi}{4}, \pm\frac{\pi}{3}, \pm\frac{\pi}{2}$

c. Plot the points in your table and sketch a graph for the equation.

5. Consider the polar equation $r = \frac{6}{3cos\theta+2sin\theta}$.

   a. Find an equivalent rectangular form for this equation.

   b. Tabulate values of r corresponding to $\theta = 0, \pm\frac{\pi}{6}, \pm\frac{\pi}{4}, \pm\frac{\pi}{3}, \pm\frac{\pi}{2}$

   c. Plot the points in your table and sketch a graph for the equation.

6. First convert to polar form and then sketch a graph for $x^2 + y^2 = (x^2 + y^2 - y)^2$

7. For the complex numbers given, find the product, $z_1 z_2$ and the quotient z_1/z_2 .

   a. $z_1 = 1 - i$ and $z_2 = \sqrt{2} - \sqrt{2}\, i$

   b. $z_1 = \sqrt{3}\left(\cos\left(\frac{\pi}{6}\right) + isin\left(\frac{\pi}{6}\right)\right)$ and $z_2 = \frac{1}{2}\left(\cos\left(\frac{2\pi}{3}\right) + isin\left(\frac{2\pi}{3}\right)\right)$

8. Find the eighth power of each complex number.

   a. $\sqrt{3}(\cos\left(\frac{\pi}{24}\right) + isin\left(\frac{\pi}{24}\right)$

   b. $\sqrt{3} - i$

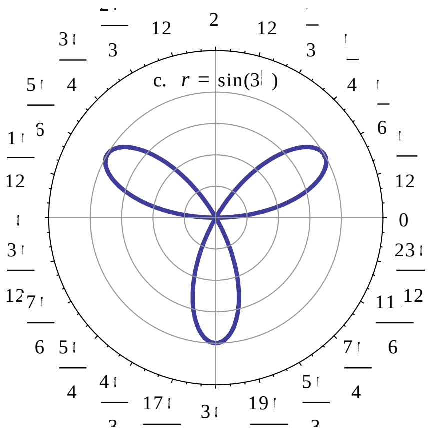
9. Find the all solutions to each equation.

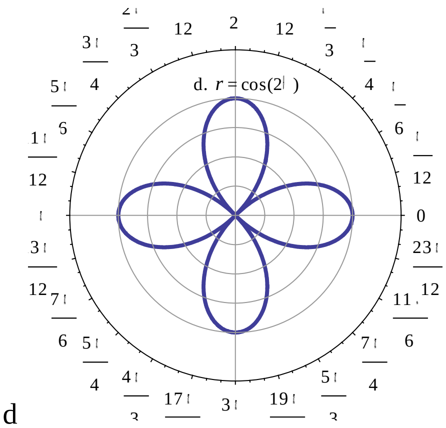
   a. $z^4 - 1 = 0$

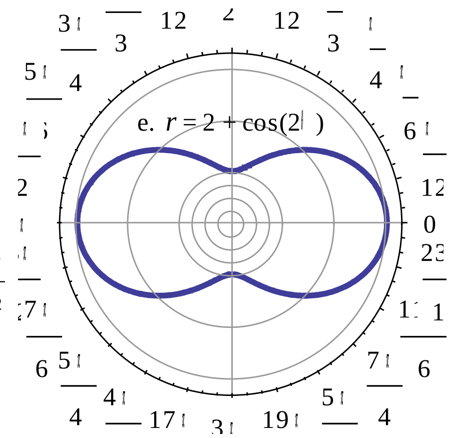
   b. $z^4 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i = 0$

Precalculus – Math 12 – fall ’09 – Test 1 Solutions.

1. Match the graph with one of the formulas.







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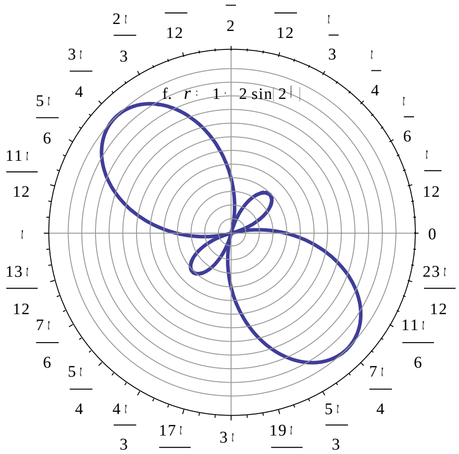
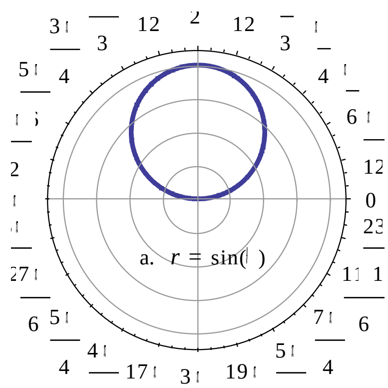
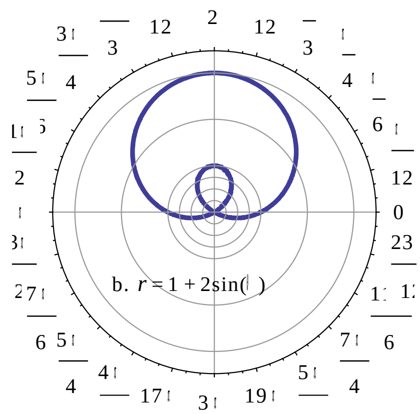






2. Convert the point whose polar coordinates are given to rectangular coordinates.

   a. $(r,\theta)=\left(2,\frac{5\pi}{4}\right)$ SOLN: $(x,y)=(r\cos\theta,r\sin\theta)=\left(2\cos\frac{5\pi}{4},2\sin\frac{5\pi}{4}\right)=\left(-\sqrt{2},-\sqrt{2}\right)$

   b. $(r,\theta)=\left(4,\frac{11\pi}{6}\right)$ SOLN: $(x,y)=(r\cos\theta,r\sin\theta)=\left(4\cos\frac{11\pi}{6},4\sin\frac{11\pi}{6}\right)=\left(2\sqrt{3},-2\right)$

3. Find two polar coordinate representations for the rectangular coordinate point, $(x,y)=\left(2,-\sqrt{3}\right)$ one with $r>0$ and one with $r<0$, and both with $0\le\theta<2\pi$.

   SOLN: We can choose $r=\pm\sqrt{(2)^2+\left(-\sqrt{3}\right)^2}=\pm\sqrt{7}$ and $\theta=\arctan\left(-\frac{\sqrt{3}}{2}\right)=-\arctan\left(\frac{\sqrt{3}}{2}\right)$ or

   $\theta=2\pi-\arctan\left(\frac{\sqrt{3}}{2}\right)$. The polar coordinates that take you to this point in QIV are thus either

   $(r,\theta)=\left(\sqrt{7},2\pi-\arctan\frac{\sqrt{3}}{2}\right)$ or $(r,\theta)=\left(-\sqrt{7},\pi-\arctan\frac{\sqrt{3}}{2}\right)$

4. Consider the polar equation $r = cos\theta + sin\theta$.

a. Find an equivalent rectangular form for this equation and write it in standard form.

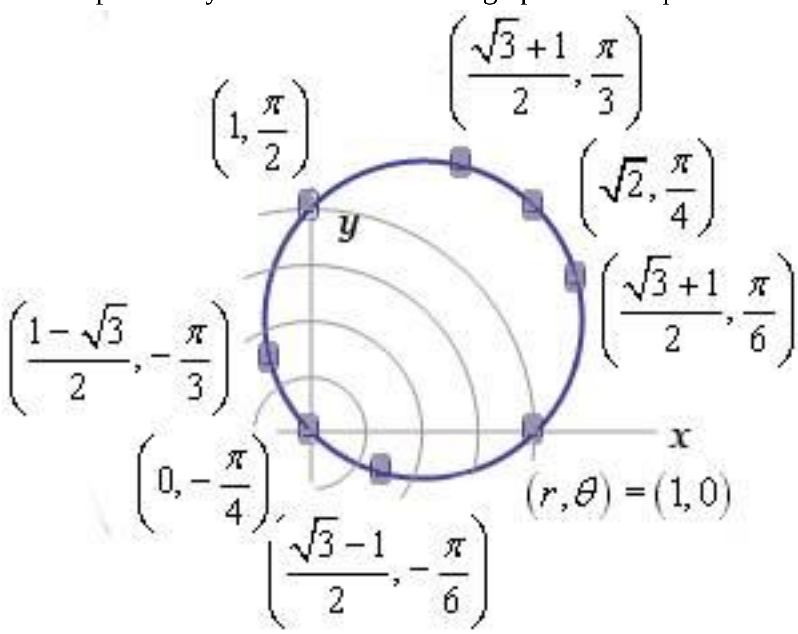
SOLN: $r = \cos\theta + \sin\theta \Rightarrow r^2 = r\cos\theta + r\sin\theta \Leftrightarrow x^2 + y^2 = x + y \Leftrightarrow \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$

b. Tabulate values of *r* corresponding to $\theta = 0, \pm\frac{\pi}{6}, \pm\frac{\pi}{4}, \pm\frac{\pi}{3}, \pm\frac{\pi}{2}$

SOLN:

r	1	$\frac{\sqrt{3}}{2} \pm \frac{1}{2} \approx 0.87 \pm 0.5$	$\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} \approx 0.7 \pm 0.7$	$\frac{1}{2} \pm \frac{\sqrt{3}}{2} \approx 0.5 \pm 0.87$	± 1
θ	0	$\pm\frac{\pi}{6}$	$\pm\frac{\pi}{4}$	$\pm\frac{\pi}{3}$	$\pm\frac{\pi}{2}$

c. Plot the points in your table and sketch a graph for the equation.



5. Consider the polar equation $r = \frac{6}{3cos\theta + 2sin\theta}$.

a. Find an equivalent rectangular form for this equation.

SOLN: $r = \frac{6}{3\cos\theta + 2\sin\theta} \Leftarrow r(3\cos\theta + 2\sin\theta) = 6 \Leftrightarrow 3r\cos\theta + 2r\sin\theta = 6 \Leftrightarrow \boxed{3x + 2y = 6}$

This is a line with slope -3/2 and y-intercept 3.

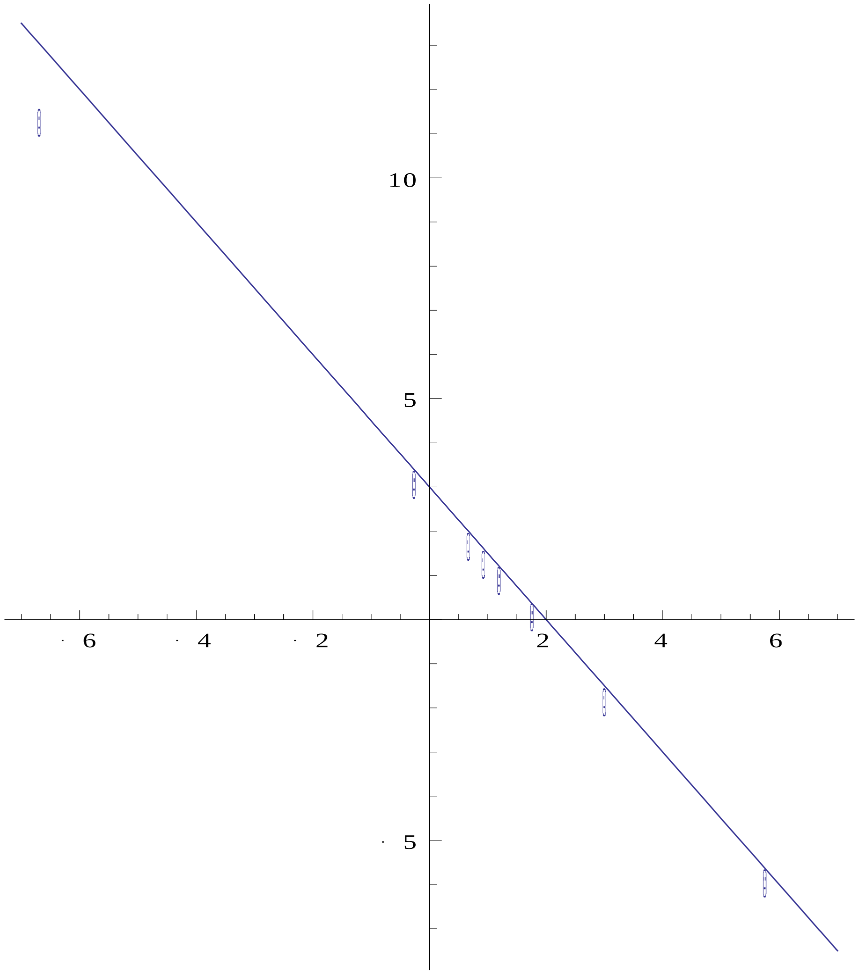
b. Tabulate values of *r* corresponding to $\theta = 0, \pm\frac{\pi}{6}, \pm\frac{\pi}{4}, \pm\frac{\pi}{3}, \pm\frac{\pi}{2}$

SOLN:

r	2	$\frac{6(3\sqrt{3} \mp 1)}{13} \approx 2.71 \mp 1.04$	$3.6\sqrt{2} \mp 2.4\sqrt{2} \approx 5.1 \mp 3.4$	$-12 \pm 8\sqrt{3} \approx -12 \pm 13.9$	± 3
θ	0	$\pm\frac{\pi}{6}$	$\pm\frac{\pi}{4}$	$\pm\frac{\pi}{3}$	$\pm\frac{\pi}{2}$

Some of these values are not so easy to approximate without a calculator, but given that we have a very simple rectangular form for the equation and the angles are fairly simple to work out where these points are.

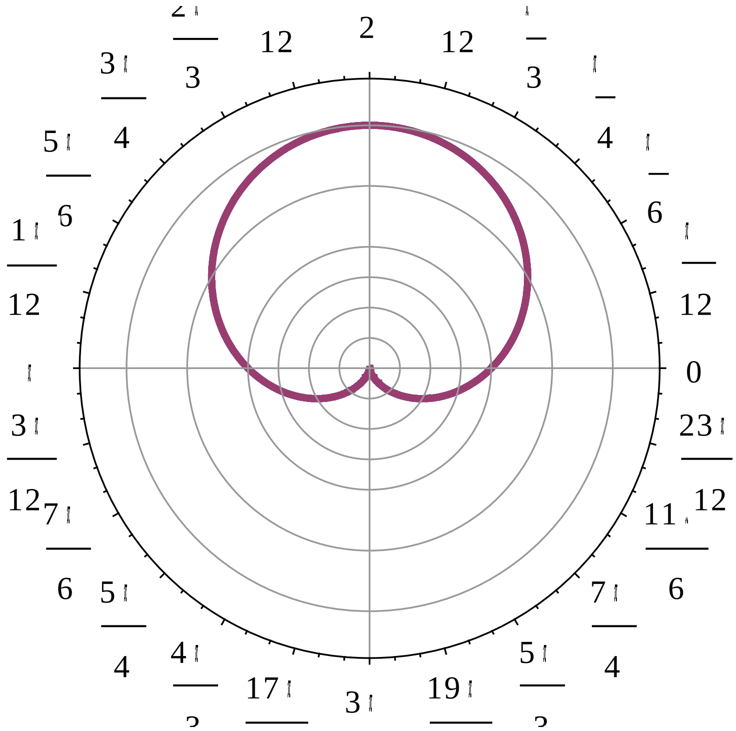
Plot the points in your table and sketch a graph for the equation.
SOLN: (Why the outlier? – I dunno…can't figure it out…)



6. First convert to polar form and then sketch a graph for $x^2 + y^2 = (x^2 + y^2 - y)^2$

$$x^2 + y^2 = \left(x^2 + y^2 - y\right)^2 \Leftrightarrow r^2 = \left(r^2 - r\sin\theta\right)^2 \Leftrightarrow r = \pm\left(r^2 - r\sin\theta\right) \Leftrightarrow \pm 1 = r - \sin\theta$$

$$\Leftrightarrow \boxed{r = \sin\theta \pm 1}$$



7. For the complex numbers given, find the product, $z_1 z_2$ and the quotient z_1/z_2 .

a. $z_1 = 1 - i$ and $z_2 = \sqrt{2} - \sqrt{2}\,i$

SOLN: $(1-i)\left(\sqrt{2}-\sqrt{2}i\right) = -2\sqrt{2}i$ and $\dfrac{1-i}{\sqrt{2}-\sqrt{2}i}\,\dfrac{\sqrt{2}+\sqrt{2}i}{\sqrt{2}+\sqrt{2}i} = \dfrac{\sqrt{2}}{2}$

b. $z_1 = \sqrt{3}\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$ and $z_2 = \frac{1}{2}\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$

SOLN: $z_1 z_2 = \dfrac{\sqrt{3}}{2}\left(\cos\left(\dfrac{5\pi}{6}\right) + i\sin\left(\dfrac{5\pi}{6}\right)\right) = \dfrac{\sqrt{3}}{2}\left(-\dfrac{\sqrt{3}}{2} + \dfrac{i}{2}\right) = -\dfrac{3}{4} + \dfrac{\sqrt{3}}{4}i$

and $\dfrac{z_1}{z_2} = 2\sqrt{3}\left(\cos\left(-\dfrac{\pi}{2}\right) + i\sin\left(-\dfrac{\pi}{2}\right)\right) = 2\sqrt{3}(0 - i) = -2\sqrt{3}i$

8. Find the eighth power of each complex number.

a. $\sqrt{3}(\cos\left(\frac{\pi}{24}\right) + i\sin\left(\frac{\pi}{24}\right)$

SOLN: $\left(\sqrt{3}\left(\cos\left(\dfrac{\pi}{24}\right) + i\sin\left(\dfrac{\pi}{24}\right)\right)\right)^8 = 81\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right) = \dfrac{81}{2} + i\dfrac{81\sqrt{3}}{2}$

b. $\left(\sqrt{3}-i\right)^8 = \left(2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)\right)^8 = 256\left(\cos\left(-\frac{4\pi}{3}\right)+i\sin\left(-\frac{4\pi}{3}\right)\right) = 256\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

$= -128+128\sqrt{3}i$

9. Find the all solutions to each equation.

a. $z^4 - 1 = 0$

SOLN: $\left(\cos\left(\frac{2\pi k}{4}\right)+i\sin\left(\frac{2\pi k}{4}\right)\right)$ for k = 0, 1, 2, 3. These come out to

$1, i, -1$ and $-i$

b. $z^4 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = 0$

SOLN $\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)^{1/4} = \left(\cos\left(-\frac{\pi}{4}+2\pi k\right)+i\sin\left(-\frac{\pi}{4}+2\pi k\right)\right)^{1/4}$

$= \boxed{\cos\left(-\frac{\pi}{16}+\frac{\pi k}{2}\right)+i\sin\left(-\frac{\pi}{16}+\frac{\pi k}{2}\right)}$

for k = 0,1,2,3