Math 12 – Precalculus – Chapt 4 Test – fall '04 NAME_____
Show all responses on separate paper. Show all work for credit.

1. Rewrite the equation $2+10^{2x-5}=10$ in equivalent logarithmic form, then solve for *x*. Finally, use a calculator to approximate *x* to the nearest thousandth.

2. Rewrite the equation $3-2\log_8(x-4)=1$ in equivalent exponential form, then solve for *x*. Do not use a calculator

3. Solve the equation for *x*. Approximate to 4 digits, if appropriate.
   a. $10^x = 6.022\times10^{23}$
   b. $1.037^{x/12} = 3$

4. Use properties of logarithms to solve the equation. Approximate to 4 digits if appropriate.
   a. $\log_2(x)+\log_2(44-3x)=7$
   b. $\log\left(\dfrac{10}{9x^2}\right)+\log(1)=0.5$

5. Given $f(x)=1+1.322^x$.
   a. Find a formula for the inverse function, $y=f^{-1}(x)$.
   b. Find the intercept and asymptote of $f(x)$.
   c. Find the intercept and asymptote of $f^{-1}(x)$.
   d. Complete the table

x	–2	0	2	4	6	8	10
$f(x)$							

   and use the results to sketch a graphing showing $y=f(x)$ and $y=f^{-1}(x)$ together and illustrating the symmetry through $y = x$.

6. Suppose the Gorkon population on planet Xorda in April of 1999 was 1380, and it is estimated that the population will increase by 2% every 400 years.
   a. Assuming a natural growth model for the Gorkons, when will its population have grown to 2000?
   b. By what percentage will the population grow in 1800 years?

7. If $300 is invested at 3.65% annual interest rate compounded daily, how long will it take to reach a value of $500?

8. Actinium has a half life of about 7.04×10^8 years. How long will it take a 1 gram sample to decay to one milligram (one thousandth of a gram).

9. Find an exponential function (of the form $f(x)=a\bullet b^x$) which passes through the points (2,5) and (5,8). What is the value of *f*(8) ?

Math 12 – Precalculus – Chapt 4 Test Solutions – fall '04

1. Rewrite the equation $2+10^{2x-5}=10$ in equivalent logarithmic form, then solve for *x*. Finally, use a calculator to approximate *x* to the nearest thousandth.

   SOLN: $2+10^{2x-5}=10 \Leftrightarrow 10^{2x-5}=8 \Leftrightarrow 2x-5=\log(8) \Leftrightarrow x=\dfrac{5+\log(8)}{2} \approx 2.952$

2. Rewrite the equation $3-2\log_8(x-4)=1$ in equivalent exponential form, then solve for *x*.

   SOLN: $3-2\log_8(x-4)=1 \Leftrightarrow \log_8(x-4)=1 \Leftrightarrow x-4=8 \Leftrightarrow x=12$

3. Solve the equation for *x*. Approximate to 4 digits, if appropriate.
   a. $10^x=6.022\times10^{23}$

      SOLN: $x=\log(6.022)+23 \approx 23.78$

   b. $1.037^{x/12}=3$

      SOLN:

      $1.037^{x/12}=3 \Leftrightarrow \ln\left(1.037^{x/12}\right)=\ln 3 \Leftrightarrow \dfrac{x}{12}\ln(1.037)=\ln 3 \Leftrightarrow x=\dfrac{12\ln 3}{\ln(1.037)} \approx 362.9$

4. Use properties of logarithms to solve the equation. Approximate to 4 digits if appropriate.
   a. $\log_2(x)+\log_2(44-3x)=7$

      $\log_2(x)+\log_2(44-3x)=7 \Rightarrow \log_2\left(x(44-3x)\right)=7 \Leftrightarrow -3x^2+44x=128$

      SOLN:

      $\Leftrightarrow 3x^2-44x-128=0 \Leftrightarrow (3x-32)(x-4)=0 \Leftrightarrow \boxed{x=\dfrac{32}{3}}$ or $\boxed{x=4}$

   b. $\log\left(\dfrac{10}{9x^2}\right)+\log(1)=0.5$

      $\log\left(\dfrac{10}{9x^2}\right)+\log(1)=0.5 \Leftrightarrow \log 10-\log 9x^2+0=0.5 \Leftrightarrow \log 9x^2=\dfrac{1}{2} \Leftrightarrow 9x^2=\sqrt{10}$

      SOLN:

      $\Leftrightarrow \boxed{x=\dfrac{\pm 10^{1/4}}{3}}$

5. Given $f(x)=1+1.322^x$.
   a. Find a formula for the inverse function, $y=f^{-1}(x)$.

      SOLN:

      $y=1+1.322^x \Leftrightarrow 1.322^x=y-1 \Leftrightarrow \log\left(1.322^x\right)=\log(y-1) \Leftrightarrow x\log 1.322=\log(y-1)$

      $x=\dfrac{\log(y-1)}{\log(1.322)} \approx 8.2487\log(y-1) \Leftrightarrow \boxed{f^{-1}(x)=8.2487\log(x-1)}$

   b. Find the intercept and asymptote of $f(x)$.

      SOLN: The graph of $y=f(x)$ rises from a horizontal asymptote *y* = 1 through the *y*-intercept at (0,2).

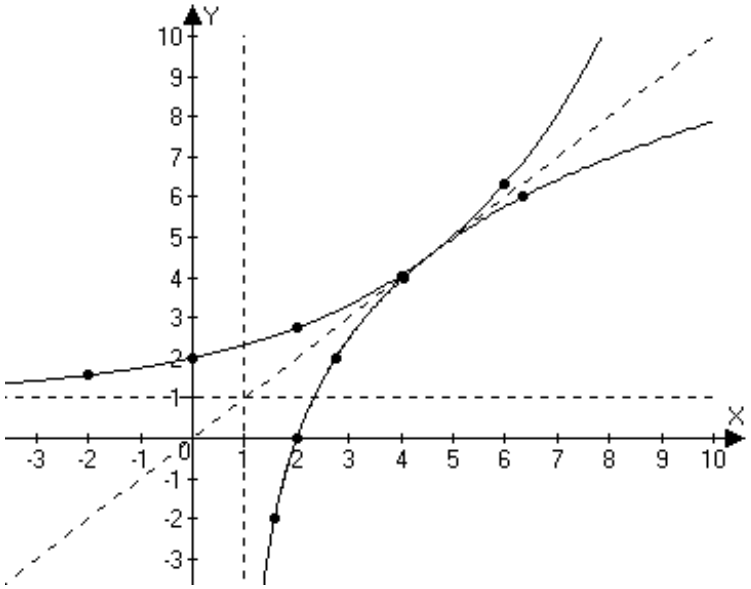
   c. Find the intercept and asymptote of $f^{-1}(x)$.

      SOLN: The graph of $y=f^{-1}(x)$ rises along a vertical asymptote *x* = 1 through the *x*-intercept at (2,0).

d. ANS: The complete the table is

x	–2	0	2	4	6	8	10
$f(x)$	1.57	2	2.75	4.05	6.34	10.3	17.3

and thus a graphing showing $y = f(x)$ and $y = f^{-1}(x)$ together illustrates the symmetry through $y = x$:



6. Suppose Tara invests $200 in a bank account paying 3% interest, compounded monthly.
   a. Write a formula which describes the amount of money in the account t years later.

   SOLN: $A(t) = 200\left(1+\frac{0.03}{12}\right)^{12t} = 200(1.0025)^{12t} \approx 200(1.030416)^{t}$

   b. How long would it take her money to double?

   SOLN: $(1.0025)^{12t} = 2 \Leftrightarrow 12t\ln(1.0025) = \ln 2 \Leftrightarrow t = \frac{\ln(2)}{12\ln(1.0025)} \approx 23.1338$ years.

   c. How long would it take her money to double if instead of being compounded monthly it was compounded continuously?

   SOLN: $e^{0.03t} = 2 \Leftrightarrow t = \frac{\ln 2}{0.03} \approx 23.1049$

7. Strontium 90 is a radioactive isotope with a half-life of 27.8 years.
   a. If 12 grams of strontium 90 are present at $t = 0$, write a formula which gives the amount (in grams) of strontium 90 which remains at time t (in years.)

   SOLN: $A(t) = 12(0.5)^{t/27.8} \approx 12(0.975375)^{t}$

   b. How long will it take the strontium 90 to decay to 2 grams?

   SOLN: $A(t) = 2 \Leftrightarrow 12(0.5)^{t/27.8} = 2 \Rightarrow (0.975375)^{t} \approx \frac{1}{6} \Rightarrow t \approx \frac{-\ln 6}{\ln 0.975375} \approx 71.86$

8. Find an exponential function of the form $P(t) = P_0 \bullet e^{rt}$ which passes through the points (0,5) and (30,10).

   SOLN: We know that $P_0 = 5$ so $P(30) = 5e^{30r} = 10 \Leftrightarrow r = \frac{\ln 2}{30} \approx 0.0231$ and thus $P(t) = 5e^{0.0231t}$