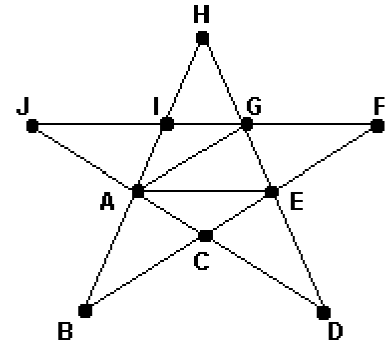
Math 13 – Spring '13 – Chapters 1-4 Test Name_________________________

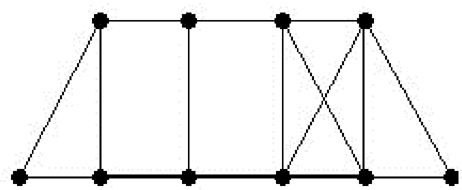
Write all responses on separate paper. Show your work for credit. Write in complete sentences.

1. Consider the graph shown to the right.
   a. List all vertices of valence 2.
   b. List all vertices of valence 3.
   c. List all vertices of valence 4.
   d. List all vertices of valence 5.
   e. Use the formula
      # edges = sum of valences/2
      to compute the number of edges.



2. For each graph, determine whether or not it has an Euler circuit. If it has an Euler circuit, indicate what it is by numbering a sequence of edges. If it doesn't have an Euler circuit, find the most efficient Eulerization by duplicating as few edges as possible.

   a.



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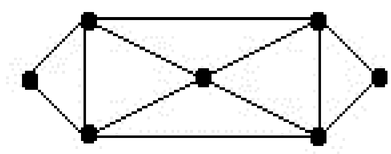
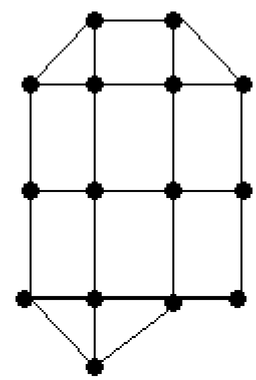


c.

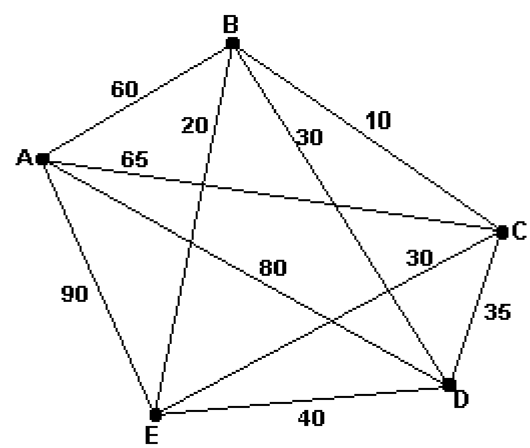


3. Is it possible to have a graph with an odd number of edges that has an euler circuit? If not, why not? If so, give an example.

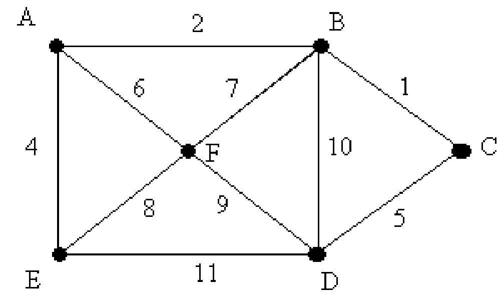
4. Consider the graph with weighted edges shown at right.

   a. What is the Hamiltonian circuit obtained by using the nearest neighbor algorithm starting at A and what does it cost?

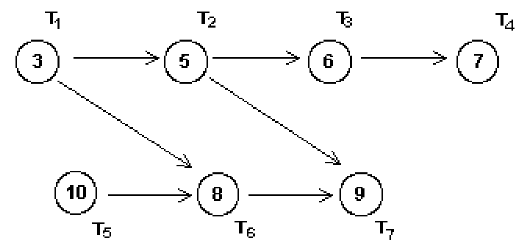
   b. What is the Hamiltonian circuit obtained by using the sorted edges algorithm?



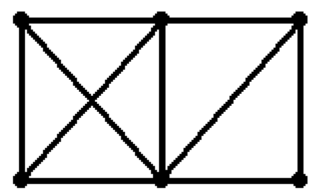
5. Consider the graph at right.
   a. Find the Hamiltonian circuit obtained by using the sorted-edges algorithm and compute its cost.
   b. Use Kruskal's sorted-edges algorithm to find a minimum-cost spanning tree for the graph. What is the cost of the minimal spanning tree?



6. Consider the order requirement digraph shown at right.
   a. What is the earliest possible completion time for a job whose order-requirement is shown at right?
   b. Use the List Processing algorithm to schedule the tasks in this digraph to two processors.



7. Find the chromatic number of the graph shown at right.



8. Use of the next-fit decreasing (NFD) bin-packing algorithm to pack objects with the following weights below into bins that can hold no more than 700 lbs.
   100 lbs, 500 lbs, 250 lbs, 350 lbs, 400 lbs, 250 lbs, 450 lbs, 200 lbs, 50 lbs

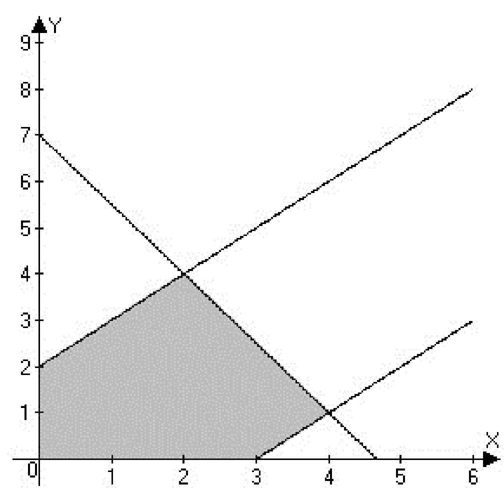
9. Graph the feasible region identified by the inequalities

$$2x + 3y \leq 12$$
$$x + 5y \leq 10$$
$$x \geq 0; \;\; y \geq 0$$

10. The feasible region for the system

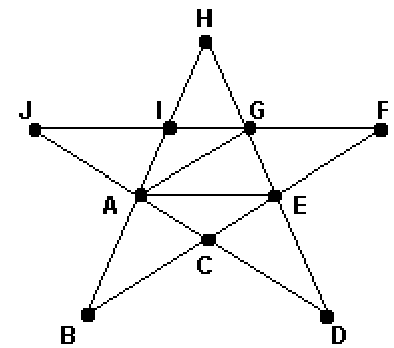
$$3x + 2y \leq 14$$
$$y - x \leq 2$$
$$y - x \geq -3$$
$$x \geq 0; \;\; y \geq 0$$

is shown at right. Find the coordinates of the corners and use these to find the point the maximizes the profit function $P = 2x + y$.



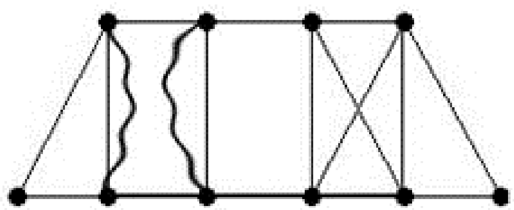
Math 13 – Spring ’13 – Chapters 1-4 Test Solutions.

1. Consider the graph shown to the right.
   a. The vertices of valence 2 are B, D, F, H and J
   b. There are no vertices of valence 3.
   c. The vertices of valence 4 are C and I.
   d. The vertices of valence 5 are E and G.
   e. Use the formula
      # edges = sum of valences/2
      to compute the number of edges.
      Since A has valence 6, the sum all valences is
      5*2+2*4+2*5+6 = 34, so there are 17 edges.

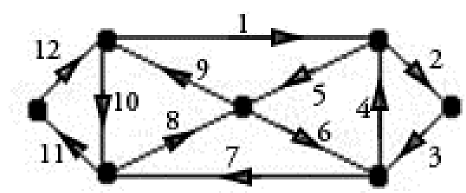


2. For each graph, determine whether or not it has an Euler circuit. If it has an Euler circuit, indicate what it is by numbering a sequence of edges. If it doesn’t have an Euler circuit, find the most efficient Eulerization by duplicating as few edges as possible.

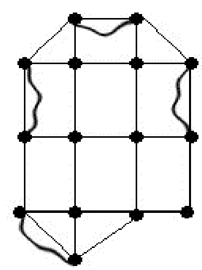
   a. At least two edges need to be duplicated.



b.

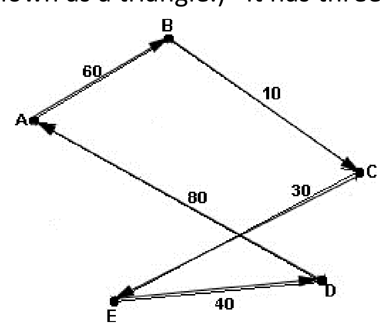


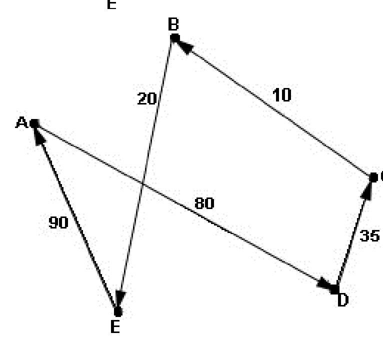
c. At least 4 edges need duplication.



3. Is it possible to have a graph with an odd number of edges that has an euler circuit? If not, why not? If so, give an example.
   ANS: Easy: Look at K_3, the complete graph on 3 vertices (also known as a triangle.) It has three

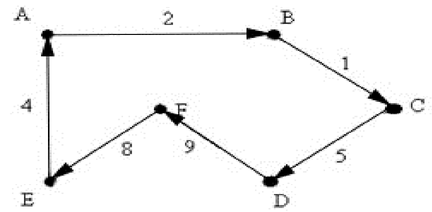
4. Consider the graph with weighted edges shown at right.
   a. What is the Hamiltonian circuit obtained by using the nearest neighbor algorithm starting at A and what does it cost? ANS: ABCEDA costs 220.
   b. What is the Hamiltonian circuit obtained by using the sorted edges algorithm?
      ANS: We add edges as follows BC, BE, CD, AD, AE to produce ADCBEA for a cost of 235.

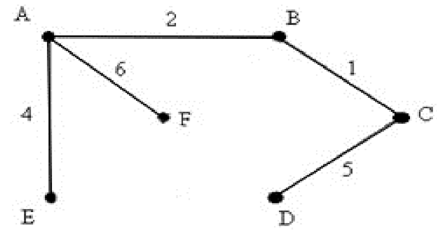




5. Consider the graph at right.

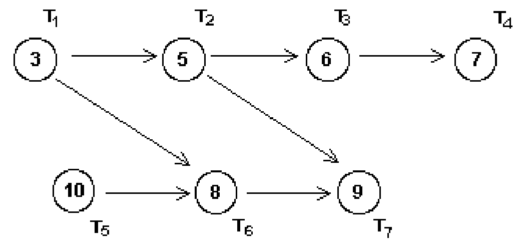
   a. Find the Hamiltonian circuit obtained by using the sorted-edges algorithm and compute its cost.
      SOLN: The luck H.C. costs 29.



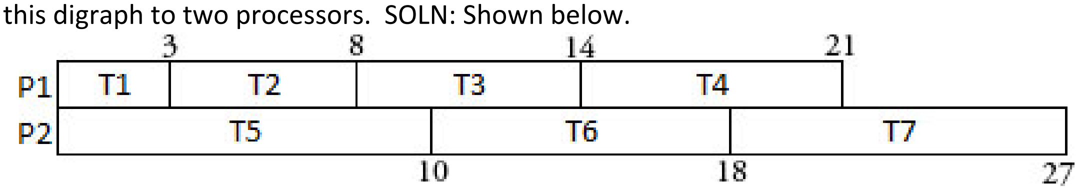


   b. Use Kruskal's sorted-edges algorithm to find a minimum-cost spanning tree for the graph. What is the cost of the minimal spanning tree?
      SOLN: The minimal spanning tree is shown to the right and has cost 18.

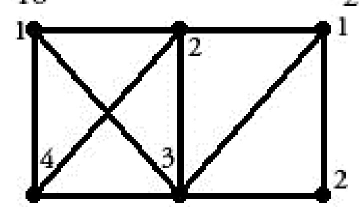
6. Consider the order requirement digraph shown at right.



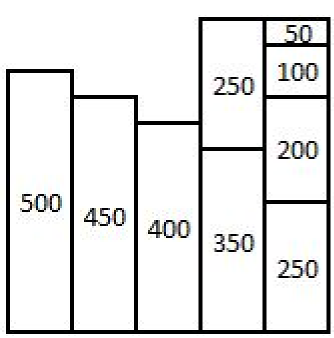
   a. What is the earliest possible completion time for a job whose order-requirement is shown at right?
      SOLN: The critical path is T5, T6, T7 and has length 27. Any schedule then will take at least 27 units of time and, with enough processors, the completion of time of 27 can be obtained.
   b. Use the List Processing algorithm to schedule the tasks in this digraph to two processors. SOLN: Shown below.



7. Find the chromatic number of the graph shown at right.
   SOLN: Note that the graph contains K_4 as a subgraph, so the chromatic number is at least 4. The labeling of vertices at right shows that 4 is enough, so the chromatic number is 4.



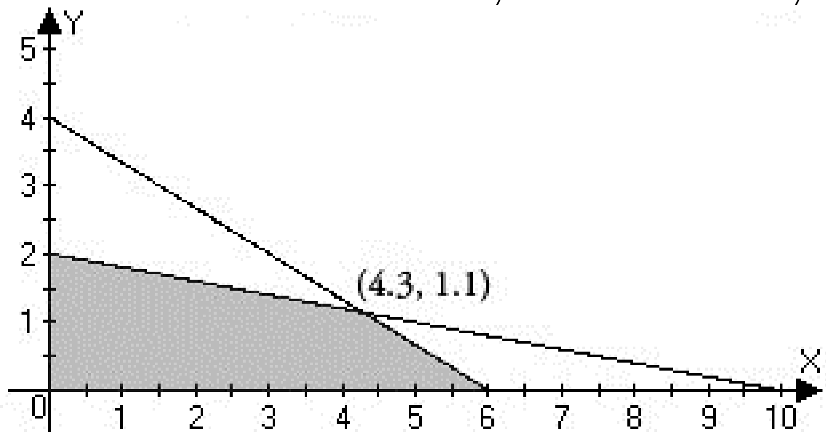
8. Use of the next-fit decreasing (NFD) bin-packing algorithm to pack objects with the following weights below into bins that can hold no more than 700 lbs.
   100 lbs, 500 lbs, 250 lbs, 350 lbs, 400 lbs, 250 lbs, 450 lbs, 200 lbs, 50 lbs
   First sort the weights in decreasing size:
   500 lbs, 450 lbs, 400 lbs, 350 lbs, 250 lbs, 250 lbs, 200 lbs, 100 lbs, 50 lbs



9. Graph the feasible region identified by the inequalities

$$2x+3y\le 12$$
$$x+5y\le 10$$
$$x\ge 0;\ \ y\ge 0$$

SOLN: The intercepts are easy, but the point of intersection is a bit tricky. Substituting $x = 10 – 5y$ from the second equation into the first yields $2(10 – 5y) + 3y = 20 – 7y = 12$ so $y=\frac{8}{7}\approx 1.1$ and $x=10-\frac{40}{7}=\frac{30}{7}\approx 4.3$



10. The feasible region for the system

$$3x+2y\le 14$$
$$y-x\le 2$$
$$y-x\ge -3$$
$$x\ge 0;\ \ y\ge 0$$

is shown at right. Find the coordinates of the corners and use these to find the point the maximizes the profit function $P=2x+y$.

The corners are (0,0), (3,0), (4,1), (2,4), and (0,2).

The respective P values are 0, 6, 9, 8, 2, so P is maximized at $P(4,1) = 9$.