Math 13 – HW 5 - Chapter 9

Write all responses on separate paper. Use complete sentences, charts and diagrams, as appropriate.

1. Explain why majority rule is not a good way to choose between four alternatives.
2. Arrow's Impossibility Theorem states that any voting system can give undesirable outcomes. Explain what this means.
3. Explain the Condorcet Winner Criterion.
4. Which of the voting procedures, plurality, the Borda count, sequential pairwise voting, or the Hare system, satisfies the Condorcet winner criterion?
5. Which of the voting procedures, plurality, the Borda count, sequential pairwise voting, or the Hare system, satisfies monotonicity?

6. In order to choose which type of music to listen to in a student center music lounge, a Condorcet vote is held by the 15 students present. Below are the preference schedules for the students. Is there a Condorcet winner and, if so, which music type?

	Number of Students		
	6	5	4
First choice	classical	rock	rock
Second choice	jazz	jazz	classical
Third choice	rock	classical	jazz

7. Given the preference schedule of 23 voters below, which candidate, if any, wins if an election is held between A and C and the winner of that race runs against B? Who wins the final election?

	Number of Voters			
	8	5	6	4
First choice	A	C	B	B
Second choice	C	A	C	A
Third choice	B	B	A	C

8. Given below is the preference schedule of 23 voters. If a Borda count is used that assigns 3 points for a first place vote, 2 points for a second place vote, and 1 point for a third place vote, who wins the election?

	Number of Voters			
	8	5	6	4
First choice	A	C	B	B
Second choice	C	A	C	A
Third choice	B	B	A	C

9. Can the four voters in the last column vote strategically to change the outcome of question 8 to one they would like better? Why or why not?

10. A 17-member committee must elect one of four candidates: R, S, T, or W. Their preference schedule is shown below. Which candidate wins under pairwise sequential voting with the agenda S, T, W, R?

	Number of Members			
	6	4	3	4
First choice	R	S	T	W
Second choice	S	R	S	T
Third choice	T	T	R	S
Fourth choice	W	W	W	R

11. There are 18 delegates to a political party's convention at which four people A, B, C, and D have been nominated as the party's candidate for governor. The delegates' preference schedule is shown below. What nominee would be elected if the party uses a rank system that assigns 5, 4, 1, and 0 point(s) for a first, second, third, and fourth choice, respectively? Is this result different from that which results from a Borda count?

	Number of Delegates		
	8	9	4
First choice	A	B	C
Second choice	B	A	B
Third choice	C	D	A
Fourth choice	D	C	D

12. An 11-member committee must choose one of the four applicants K, L, M, and N for membership on the committee. The committee members have preferences among the applicants as shown below. Which applicant will be given the position if the members use the Borda count to choose the new member?

	Number of Members		
	6	2	3
First choice	K	M	M
Second choice	L	L	N
Third choice	N	K	L
Fourth choice	M	N	K

13. Consider the following preference table:

	Number of voters			
	4	6	8	4
First choice	D	C	A	B
Second choice	C	B	D	A
Third choice	B	D	C	C
Fourth choice	A	A	B	D

Which candidate will be chosen if the Borda count is used?

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Write all responses on separate paper. Use complete sentences, charts and diagrams, as appropriate.

1. Explain why majority rule is not a good way to choose between four alternatives.
   SOLN: Majority rule is not a good way to choose among four alternatives because it is possible that none of the four will get a majority of the vote.
2. Arrow's Impossibility Theorem states that any voting system can give undesirable outcomes. Explain what this means.
   SOLN : For any voting system it is possible to find a set of voter's preferences that will cause the voting system to violate a condition deemed desirable for a fair voting system. These conditions may include the Condorcet Winner Criterion and the Independence of Irrelevant Alternatives condition.
3. Explain the Condorcet Winner Criterion.
   SOLN: A voting system satisfies the Condorcet Winner Criterion if the winner of an election is also the Condorcet winner, if a Condorcet winner exists.
4. Which of the voting procedures, plurality, the Borda count, sequential pairwise voting, or the Hare system, satisfies the Condorcet winner criterion?
   SOLN: Only sequential pairwise voting satisfies the Condorcet Winner Criterion.
5. Which of the voting procedures, plurality, the Borda count, sequential pairwise voting, or the Hare system, satisfies monotonicity?
   SOLN: Plurality, the Borda count, and sequential pairwise voting all satisfy monotonicity.

6. In order to choose which type of music to listen to in a student center music lounge, a Condorcet vote is held by the 15 students present. Below are the preference schedules for the students. Is there a Condorcet winner and, if so, which music type?

	Number of Students		
	6	5	4
First choice	classical	rock	rock
Second choice	jazz	jazz	classical
Third choice	rock	classical	jazz

   SOLN: Rock is the Condorcet winner.
7. Given the preference schedule of 23 voters below, which candidate, if any, wins if an election is held between A and C and the winner of that race runs against B? Who wins the final election?

	Number of Voters			
	8	5	6	4
First choice	A	C	B	B
Second choice	C	A	C	A
Third choice	B	B	A	C

   SOLN: There is no majority-rule winner.

8. Given below is the preference schedule of 23 voters. If a Borda count is used that assigns 3 points for a first place vote, 2 points for a second place vote, and 1 point for a third place vote, who wins the election?

	Number of Voters			
	8	5	6	4
First choice	A	C	B	B
Second choice	C	A	C	A
Third choice	B	B	A	C

SOLN: A wins.

9. Can the four voters in the last column vote strategically to change the outcome of question 8 to one they would like better? Why or why not?
SOLN: No. They cannot make B win the election and if they switch any rankings to place C higher, then C would win and this is their least desirable outcome.

10. A 17-member committee must elect one of four candidates: R, S, T, or W. Their preference schedule is shown below. Which candidate wins under pairwise sequential voting with the agenda S, T, W, R?

	Number of Members			
	6	4	3	4
First choice	R	S	T	W
Second choice	S	R	S	T
Third choice	T	T	R	S
Fourth choice	W	W	W	R

SOLN: S wins.

11. There are 18 delegates to a political party's convention at which four people A, B, C, and D have been nominated as the party's candidate for governor. The delegates' preference schedule is shown below. What nominee would be elected if the party uses a rank system that assigns 5, 4, 1, and 0 point(s) for a first, second, third, and fourth choice, respectively? Is this result different from that which results from a Borda count?

	Number of Delegates		
	8	9	4
First choice	A	B	C
Second choice	B	A	B
Third choice	C	D	A
Fourth choice	D	C	D

SOLN: With the given rank system, candidate B wins. Under a Borda Count, candidate B also wins.

12. An 11-member committee must choose one of the four applicants K, L, M, and N for membership on the committee. The committee members have preferences among the applicants as shown below. Which applicant will be given the position if the members use the Borda count to choose the new member?

	Number of Members		
	6	2	3
First choice	K	M	M
Second choice	L	L	N
Third choice	N	K	L
Fourth choice	M	N	K

SOLN: Applicant K is chosen.

13. Consider the following preference table:

	Number of voters			
	4	6	8	4
First choice	D	C	A	B
Second choice	C	B	D	A
Third choice	B	D	C	C
Fourth choice	A	A	B	D

Which candidate will be chosen if the Borda count is used?

SOLN: C