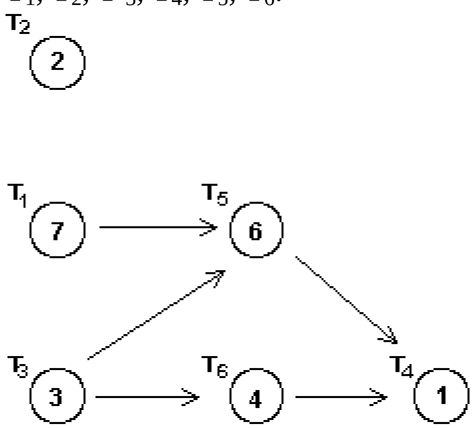
Math 13 – Liberal Arts Math – HW3 - Spring ’13 Name___________________________________
HW 3 – Write all responses on separate paper. Use complete sentences where appropriate.

1. Consider the problem of packing following list of weights into bins that can hold no more than 9 lbs.
   5 lbs, 7 lbs, 1 lb, 2 lbs, 4 lbs, 5 lbs, 1 lb, 1 lb, 3 lbs, 6 lbs, 2 lbs
   a. Use the first fit (FF) bin-packing algorithm to pack the weights into these bins.
   b. Use the next fit (NF) bin-packing algorithm to pack weights into these bins.
   c. Use the worst fit (WF) bin-packing algorithm to pack the weights into these bins.
   d. Use the first-fit decreasing (FFD) bin-packing algorithm to pack the weights into these bins.
   e. Use the next-fit decreasing (NFD) bin-packing algorithm to pack the weights into these bins.
   f. Use the worst-fit decreasing (WFD) bin-packing algorithm to pack the weights into these bins.

2. Given the order-requirement digraph below (with time given in minutes) and the priority list
   $T_1, T_2, T_3, T_4, T_5, T_6$.



   a. Apply the list-processing algorithm to construct a schedule using two processors.
   b. Apply the list-processing algorithm to construct a schedule using three processors.
   c. Apply the critical-path scheduling algorithm to construct a schedule using two processors.
   d. Apply the critical-path scheduling algorithm to construct a schedule using three processors.

3. What is the minimum time required to complete 12 independent tasks on three processors when the sum of all the times of the 12 tasks is 60 minutes? Explain.

4. What is the minimum time required to complete eight independent tasks on two processors when the sum of the times of the eight tasks is 64 minutes? Explain.

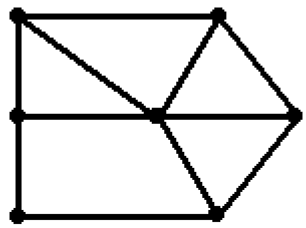
5. Give an example of an order-requirement digraph with six tasks $T_1, T_2, T_3, T_4, T_5, T_6$ for which the critical-path is T_1, T_3, T_4.

6. Give an example in which the first fit (FF) and next fit (NF) bin-packing algorithms produce the same packing.

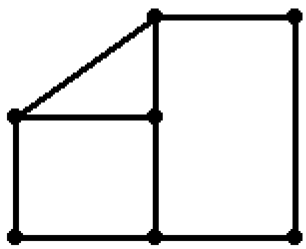
7. Why are there several different algorithms for the bin-packing problem? Why not just use the algorithm that works best?

8. When scheduling tasks using an order-requirement digraph, why does the critical-path scheduling algorithm generally produce good schedules? Does it always produce an optimal schedule?

9. Find the chromatic number of the graph below



10. Find the chromatic number of the graph below.

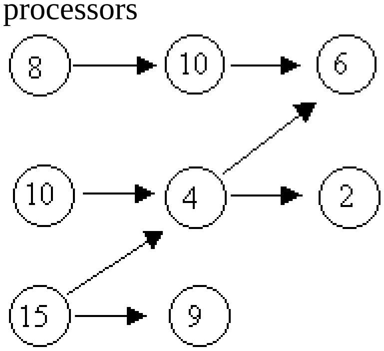


11. The table below represents species of plants that have competing light or water requirements. Draw the graph which would be useful in determining the minimum number of different habitats that would be needed to display all these plants in a garden

	A	B	C	D	E
A		X		X	X
B	X		X		
C		X		X	
D	X		X		X
E	X			X	

12. Use the decreasing-time list algorithm to schedule these tasks on three machines
7, 2, 5, 3, 9, 1, 6, 5, 3, 7
How much time does the resulting schedule require?

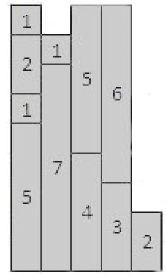
13. Given the order requirement digraph below (with time given in minutes) and the priority list T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, apply the critical-path scheduling algorithm to construct a schedule using two processors



Math 13 – Liberal Arts Math – HW3 Solutions

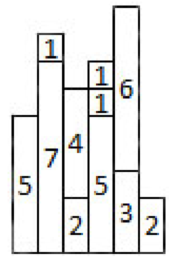
1. Consider the problem of packing following list of weights into bins that can hold no more than 9 lbs.
   5 lbs, 7 lbs, 1 lb, 2 lbs, 4 lbs, 5 lbs, 1 lb, 1 lb, 3 lbs, 6 lbs, 2 lbs

   a. Use the first fit (FF) bin-packing algorithm to pack the weights into these bins.
   SOLN: (shown at right)

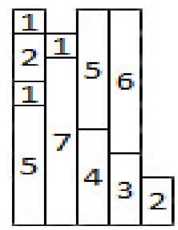


   b. Use the next fit (NF) bin-packing algorithm to pack weights into these bins.
   SOLN:

```
        1
  1  4  1  6
5  7  2  5  3  2
```

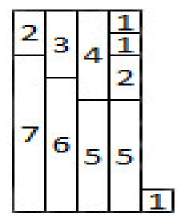


   c. Use the worst fit (WF) bin-packing algorithm to pack the weights into these bins.
   SOLN:
   1
   2
   f 1 5 6
   5 7 4 3 2

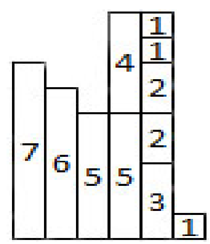


   d. Use the first-fit decreasing (FFD) bin-packing algorithm to pack the weights into these bins.

```
          1
          1
2  3  4   2
7  6  5   5   1
```

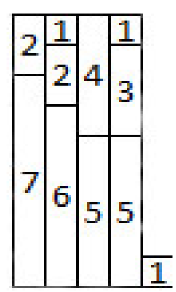


   e. Use the next-fit decreasing (NFD) bin-packing algorithm to pack the weights into these bins.

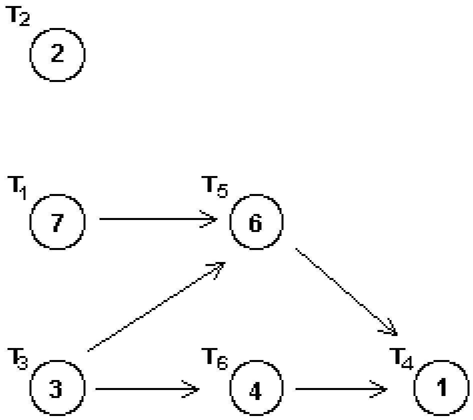


   f. Use the worst-fit decreasing (WFD) bin-packing algorithm to pack the weights into these bins.

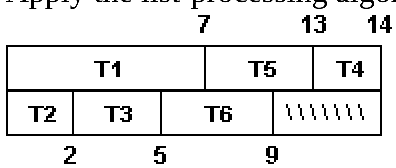
```
   1       1
2  2   4   3
7  6   5   5   1
```



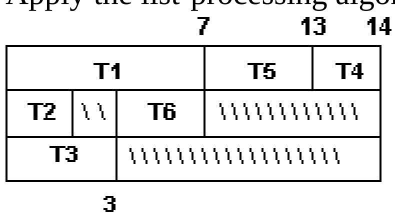
2. Given the order-requirement digraph below (with time given in minutes) and the priority list T_1, T_2, T_3, T_4, T_5, T_6.



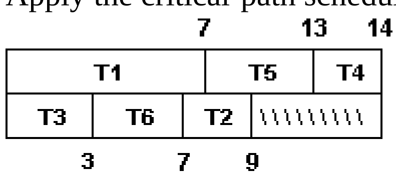
a. Apply the list-processing algorithm to construct a schedule using two processors.



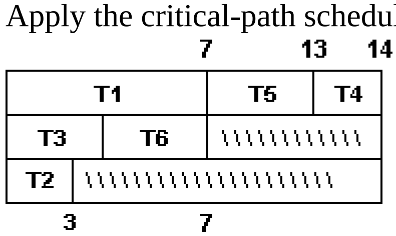
b. Apply the list-processing algorithm to construct a schedule using three processors.



c. Apply the critical-path scheduling algorithm to construct a schedule using two processors.



d. Apply the critical-path scheduling algorithm to construct a schedule using three processors.



3. What is the minimum time required to complete 12 independent tasks on three processors when the sum of all the times of the 12 tasks is 60 minutes? Explain.
   ANS: The most efficient time is the sum of the tasks' times/(# processors) = 60/3 = 20 minutes.
4. What is the minimum time required to complete eight independent tasks on two processors when the sum of the times of the eight tasks is 64 minutes? Explain.

ANS: Same idea: the most efficient time is the sum of the tasks' times/2 = 64/2 = 32 minutes

5. Give an example of an order-requirement digraph with six tasks T_1, T_2, T_3, T_4, T_5, T_6 for which the critical-path is T_1, T_3, T_4.
ANS: For instance,

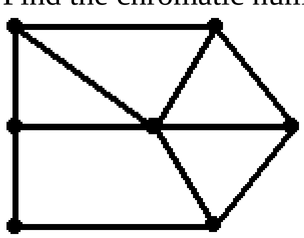
$$\begin{array}{l} T_1\ (5) \rightarrow T_3\ \ (6) \rightarrow T_4\ (5) \\ \ \ \ \uparrow \\ T_2\ (1) \rightarrow T_5\ \ (1) \rightarrow T_6\ (1) \end{array}$$

6. Give an example in which the first fit (FF) and next fit (NF) bin-packing algorithms produce the same packing.
ANS: For example, when the capacity is 10 lbs. The weights are 7 lbs, 6 lbs, 5 lbs, and 5 lbs.

7. Why are there several different algorithms for the bin-packing problem? Why not just use the algorithm that works best?
ANS: There are different situations and no best algorithm.

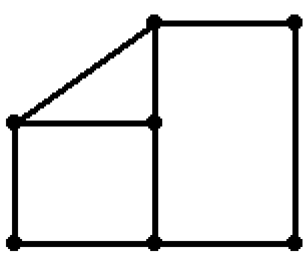
8. When scheduling tasks using an order-requirement digraph, why does the critical-path scheduling algorithm generally produce good schedules? Does it always produce an optimal schedule?
ANS: It produces good schedules because the bigger tasks are done early, allowing for smaller tasks to fill in the remaining time.

9. Find the chromatic number of the graph below



ANS: The chromatic number is 3.

10. Find the chromatic number of the graph below.

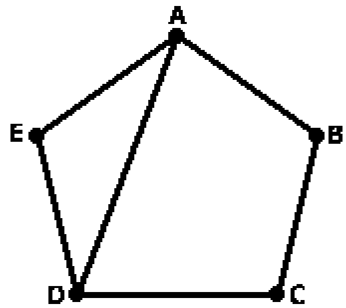


The chromatic number is 3.

11. The table below represents species of plants that have competing light or water requirements. Draw the graph which would be useful in determining the minimum number of different habitats that would be needed to display all these plants in a garden

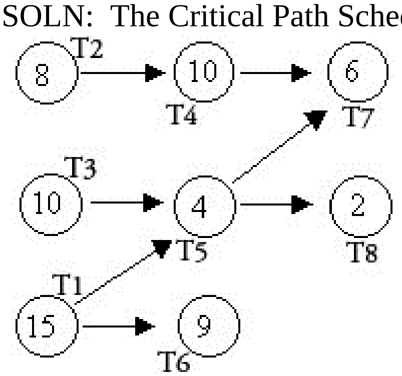
	A	B	C	D	E
A		X		X	X
B	X		X		
C		X		X	
D	X		X		X
E	X			X	

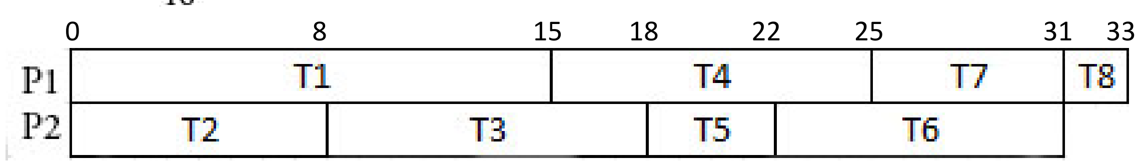
ANS: One solution is this graph:



12. Use the decreasing-time list algorithm to schedule these tasks on three machines
7, 2, 5, 3, 9, 1, 6, 5, 3, 7
How much time does the resulting schedule require?
ANS: 16 minutes

13. Given the order requirement digraph below (with time given in minutes) and the priority list T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, apply the critical-path scheduling algorithm to construct a schedule using two processors
SOLN: The Critical Path Scheduling Algorithm leads to the tasks labeled as shown below:





For comparison, here's how it looks with three processors.