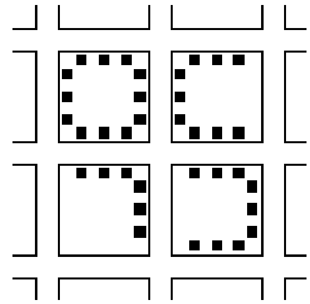
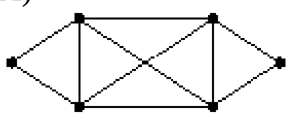
Math 13 —Review Problems for Final Exam— Spring 2013
If this were a test, the instructions would be to write all responses on separate paper. Show your work for credit. Explain your answers using complete sentences.



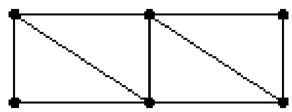
1. The map shown to the right illustrates part of a meter-reader's territory. The squares represent homes with meters to be read. Draw a graph that would be used for finding an efficient route past each of the homes. Assume the reader can check meters on only one side of a street at a time.

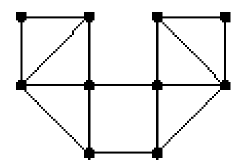
2. For each graph below, either prove that it has an Euler circuit or duplicate an minimal number of edges to Eulerize it.

A)

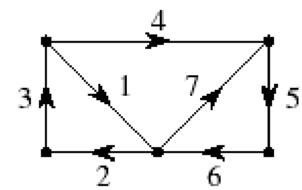


B)

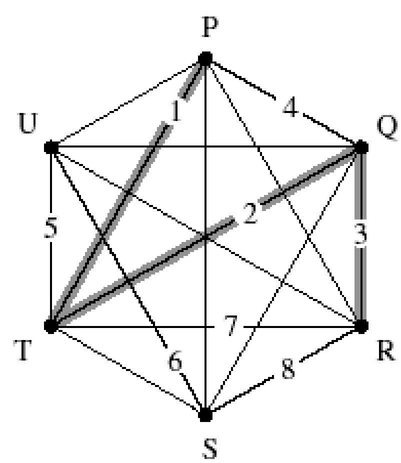




3. In order to Eulerize the graph on the right, what is the fewest number of edges that need to be added or duplicated?



4. Consider the path represented by the numbered sequence of edges on the graph to the right.
A) Is the path is an Euler circuit?
B) Is the path is a Hamiltonian circuit?



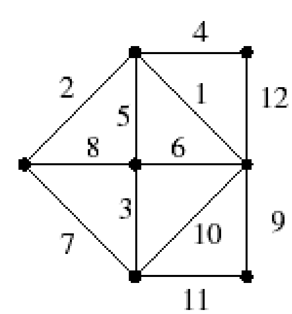
Questions 5-8 relate to the figure on the right. The numbers placed on top of edges are weights. Assume that edges with **no** indicated weight (those drawn with the light lines) have weight at least 10, and so can be ignored for these questions. In each case, use the given algorithm to pick the **next** edge to be added from the five choices below, where the first three edges, marked with (PT, TQ, QR), have **already** been selected.

5. Nearest-neighbor algorithm starting at P

6. Sorted-edges algorithm

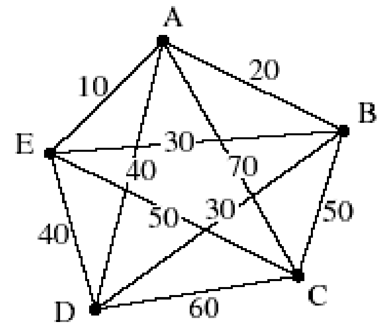
7. Kruskal's algorithm

8. In What is the minimal Hamiltonian circuit assuming that the path starts with PTQR?



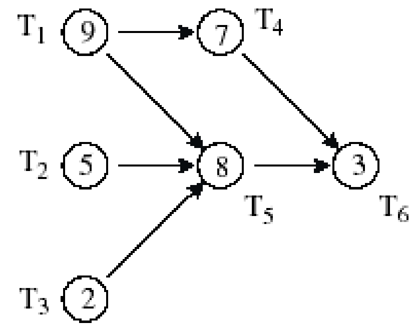
9. Find the minimum-cost spanning tree obtained from Kruskal's algorithm on the graph to the right:

10. For the graph on the right, what is the cost of the Hamiltonian circuit obtained by using the nearest-neighbor algorithm, starting at A?



11. For the graph on the right, (the same as in 11) what is the cost of the Hamiltonian circuit obtained by using the sorted-edges algorithm?

Questions 12-14 refer to the order-requirement digraph at right.

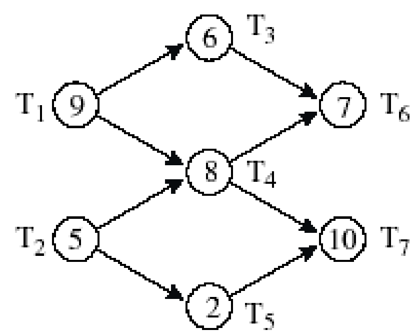


12. What is the earliest possible completion time for a job with this order-requirement digraph?

13. Suppose a crew can complete a job with this order-requirement digraph in a minimum amount of time. If task T_5 is shortened from 8 min to 5 min, what is the maximum amount by which the completion time of the entire job can be shortened?

14. Given the above digraph and the priority list T_1, T_2, …, T_6, use the list-processing algorithm to construct a schedule using 2 processors. How much time does the resulting schedule take?

Questions 15-16 relate to the order-requirement digraph on the right and the schedule obtained from the **critical-path scheduling** algorithm using **two** processors.



15. Find the resulting priority list.

16. Write out the schedule.

Questions 17-20 follow a common thread and should be read in order.

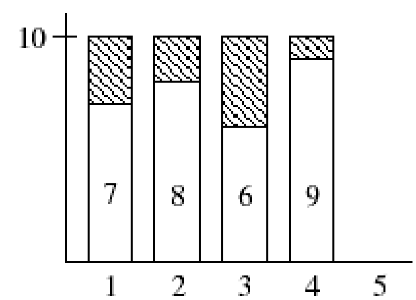
17. A caterer has two ovens in which she must prepare 15 dishes for an upcoming event. The dishes will bake for differing amounts of time and the quantities required are such that each dish will exactly fill an oven during the time it is baking. All of the dishes can sit after baking before serving, so there are no constraints on the order in which the dishes are baked. The caterer wishes to complete the baking in as short a period of time as possible. What technique most likely to be useful in solving this problem? Finding an Euler circuit on a graph? Using the decreasing-time-list algorithm? Applying the sorted-edges algorithm? Using the worst-fit algorithm? Applying Kruskal's algorithm?

18. The caterer (see #17) decides that the time required to bake all of the dishes in just two ovens is too long. She feels that in order for the quality of the food not to suffer, the baking must all take place within a 3 hour span. She will rent additional ovens to accomplish this, but she wishes to rent only as many as absolutely necessary. What technique most likely to be useful in solving this problem? Finding an Euler circuit on a graph? Using the decreasing-time-list algorithm? Applying the sorted-edges algorithm? Using the worst-fit algorithm? Applying Kruskal's algorithm? Explain.

19. Our caterer (see #17, 18) is doing all of the food preparation at the convention center in which the event will take place. There are not enough electric circuits in the center's kitchen to support all of the additional rental ovens. Therefore, the ovens are spread among 5 locations within the convention center. The caterer must repeatedly travel to each location to check the progress of the baking. She needs a route that will take her to each of the other locations and back to the kitchen as efficiently as possible. What technique most likely to be useful in solving this problem? Finding an Euler circuit on a graph? Using the decreasing-time-list algorithm? Applying the sorted-edges algorithm? Using the worst-fit algorithm? Applying Kruskal's algorithm? Explain.

20. Not following your advice to #19, the caterer took a different route each time she went around checking the ovens. She realized at the end that at some point she dropped her favorite chef's hat somewhere in the halls of the convention center. She wishes to rewalk, as efficiently as possible, each of the halls she had been through earlier in an attempt to find the hat. What technique most likely to be useful in solving this problem? Finding an Euler circuit on a graph? Using the decreasing-time-list algorithm? Applying the sorted-edges algorithm? Using the worst-fit algorithm? Applying Kruskal's algorithm? Explain.

Questions 21-23 relate to the following problem. Suppose in a binpacking problem, the first 4 weights have already been placed into bins, as shown in the diagram on the right. Each bin has capacity 10. The next (fifth) weight to be placed has size 2. In questions 21-23, use the given algorithm to decide which bin this weight should be placed.



21. Where should a weight of size 2 be placed using the worst-fit (WF) algorithm?

22. Where should a weight of size 2 be placed using the first-fit (FF) algorithm?

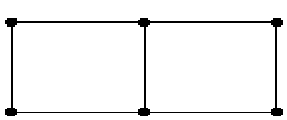
23. Where should a weight of size 2 be placed using the next-fit (NF) algorithm?

24. The table to the right shows pairs of chemical compounds (×) that cannot be mixed without causing a dangerous reaction. Draw a graph which would be used to schedule disposal containers for the compounds. How many containers would be required?

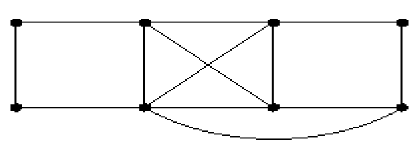
	A	B	C	D	E	F
A			×	×	×	
B			×		×	×
C	×	×		×		×
D	×		×			×
E	×	×				
F		×	×	×		

25. Find the chromatic number of the graphs below.

A)



B)



26. A cheeseburger requires 5oz of meat and 0.7oz of cheese, while a superburger requires 7 oz of meat and 0.6oz of cheese. A burger stand has 350oz of meat and 42oz of cheese available. The profit on a cheeseburger is 10 cents and on a superburger is 40 cents. Let x represent the number of cheeseburgers produced and y the number of superburgers produced.

A) What are the constraint inequalities for this situation?

B) What is the objective function?

C) How many cheeseburgers and superburgers should be produced to maximize profit?

27. Amazin' Raisin Baking Co. makes both raisin cakes and raisin pies. A cake requires 25 oz of raisins and 5 oz sugar. A pie requires 20 oz of raisins and 10 oz of sugar. There are 500 oz of raisins available and 160 oz of sugar. The profit on one cake is $5 and on one pie is $6. How much of each should they make in order to maximize profit?

28. Using the preference lists on the right, calculate the winner of an election by

A) plurality voting?
B) the Hare system?
C) a Borda count?
D) pairwise sequential voting with agenda BDECA.

# of voters:	7	5	5	3
Rankings:	*B*	*C*	*A*	*A*
	C	*E*	*E*	*D*
	E	*B*	*D*	*E*
	A	*D*	*B*	*C*
	D	*A*	*C*	*B*

29. Is there a Condorcet winner for the preference lists on the right? If so, who is it?

# of voters:	4	3	2
Rankings:	*A*	*C*	*D*
	B	*B*	*B*
	C	*D*	*A*
	D	*A*	*C*

30. Calculate the Banzhaf Power Indices for the weighted voting system [11: 8, 4, 2, 1].

31. Calculate the Shapley-Shubik Power Indices for the weighted voting system [13: 7, 6, 5, 3].

Also fair game are binary linear codes, Huffman compression, cryptography, Fibonacci-type sequences, rosettes, friezes, wallpaper, and tiling.