Math13 – HW 6 – Chapter 10

1. If a voting system has three or more alternatives, satisfies the Pareto condition, always produces a unique winner, and is not a dictatorship, what conclusion follows from the GS theorem?
2. Are there voting methods that are never manipulable? Give an example.
3. A 17-member committee must elect one of four candidates: R, S, T, or W. In a plurality runoff election candidate S wins.

	Number of Members			
	6	4	3	4
First choice	R	S	T	W
Second choice	S	R	S	T
Third choice	T	T	R	S
Fourth choice	W	W	W	R

   a. Could those members who most prefer T vote strategically in some way to change the outcome in a way that will benefit them?
   b. Could those members who most prefer S vote strategically in some way to change the outcome in a way that will benefit them?
   c. Is it possible to manipulate the results of a sequencial pairwise election?
   d. What would happen if the four voters who prefer W insincerely voted for T instead? Is this in their best interests?
   e. What would happen if the four voters who prefer T insincerely voted for S instead? Is this in their best interests?
4. An 11-member committee must choose one of the four applicants K, L, M, and N for membership on the committee. The committee members have preferences among the applicants as given below.

	Number of Members		
	6	2	3
First choice	K	M	M
Second choice	L	L	N
Third choice	N	K	L
Fourth choice	M	N	K

   a. If the committee uses pairwise sequential voting with the agenda K, L, M, N, applicant K wins. Can the three voters who least prefer K vote strategically in some way to change the outcome to one they find more favorable? Why or why not?
   b. If the committee uses pairwise sequential voting with the agenda K, L, M, N, applicant K wins. Is it possible that another agenda will yield a different winner?
   c. Suppose the Borda Count method is used. Who wins the election? Can the group of three voters favorably impact the results through insincere voting?
   d. Suppose the Borda Count method is used. Can the group of two voters favorably impact the results through insincere voting?
   e. Suppose the Borda Count method is used. Suppose the group of six suspect that the group of two intends to insincerely exchange M and L in their rankings. Can the group of six counteract in order to protect K as the winner?
5. Consider an 11-member committee that must choose one of three alternatives: X, Y, or Z, using the Hare system. Their schedule of preferences is shown below. Who wins?

	Number of Voters		
	5	4	2
First choice	Z	X	Y
Second choice	Y	Y	X
Third choice	X	Z	Z

a. Is it possible for the group of five voters to change the outcome in a way that would benefit them?
b. Is it possible for the group of two voters to change the outcome in a way that would benefit them?
c. However, the committee suspects that the group of five plans to insincerely reorder their preferences as Y, Z, X. How can the group of four respond?

6. A group of 22 young people must decide whether to go to the beach (B), the mountain (M), or the zoo (Z) on a field trip. Their preference rankings are summarized in the table below, and the decision will be made using a Borda count. Who wins the vote?

	Number of Voters		
	10	8	4
First choice	B	M	Z
Second choice	M	B	M
Third choice	Z	Z	B

a. Can the four voters in the last column change the results of the vote to their favor by changing their preference rankings?
b. Can the 10 voters in the first column change the results of the vote to their favor by changing their preference rankings?

7. Consider the following preference table:

	Number of voters			
	4	6	8	4
First choice	D	C	A	B
Second choice	C	B	D	A
Third choice	B	D	C	C
Fourth choice	A	A	B	D

a. Who wins using plurality? Could the four voters who most prefer B vote insincerely to change the outcome in a way that would benefit them?
b. Could the six voters who most prefer C vote insincerely to change the outcome in a way that would benefit them?

Math13 – HW 6 Solutions – Chapter 10

1. If a voting system has three or more alternatives, satisfies the Pareto condition, always produces a unique winner, and is not a dictatorship, what conclusion follows from the GS theorem?
   SOLN: The voting method can be manipulated,
2. Are there voting methods that are never manipulable? Give an example.
   SOLN: Yes, Condorcet's Method for example
3. A 17-member committee must elect one of four candidates: R, S, T, or W. In a plurality runoff election candidate S wins.

	Number of Members			
	6	4	3	4
First choice	R	S	T	W
Second choice	S	R	S	T
Third choice	T	T	R	S
Fourth choice	W	W	W	R

   a. Could those members who most prefer T vote strategically in some way to change the outcome in a way that will benefit them?
      SOLN: Yes, they could swap their first and second ranked candidates, then S would beat R by 7 to 6, a preferred outcome for those voters.
   b. Could those members who most prefer S vote strategically in some way to change the outcome in a way that will benefit them?
      SOLN: No, those voters prefer R to T or W and R is the plurality winner, so they can't do better than that.
   c. Is it possible to manipulate the results of a sequential pairwise election?
      SOLN: No. S will always win.
      There are 24 permutations of the candidates:
      RSTW has S beating R, then S beating T, then S beating W.
      This shows that S is the Condorcet winner, and so S will be the winner in any agenda.
   d. What would happen if the four voters who prefer W insincerely voted for T instead? Is this in their best interests?
      SOLN: T and R would face off in the runoff, and R would win. In this case, their least favorite candidate would win instead of a higher ranked alternative.
   e. What would happen if the four voters who prefer T insincerely voted for S instead? Is this in their best interests?
      SOLN: S and R would face off in the runoff, but S would again win. They cannot force a win for their first choice, but they can show allegiance to their second choice and eventual winner.
4. An 11-member committee must choose one of the four applicants K, L, M, and N for membership on the committee. The committee members have preferences among the applicants as given below.

	Number of Members		
	6	2	3
First choice	K	M	M
Second choice	L	L	N
Third choice	N	K	L
Fourth choice	M	N	K

   a. If the committee uses pairwise sequential voting with the agenda K, L, M, N, applicant K wins. Can the three voters who least prefer K vote strategically in some way to change the outcome to one they find more favorable? Why or why not?
      SOLN: No. The six voters who most prefer applicant K represent a majority of the committee. No matter how the three voters rank the applicants, K will win.

b. If the committee uses pairwise sequential voting with the agenda K, L, M, N, applicant K wins. Is it possible that another agenda will yield a different winner?
SOLN: No. The six voters who most prefer applicant K represent a majority of the committee. No matter how the voters are ordered, K will win.

c. Suppose the Borda Count method is used. Who wins the election? Can the group of three voters favorably impact the results through insincere voting?
SOLN: K currently wins. Yes. For example, by exchanging L and N, L will win instead.

d. Suppose the Borda Count method is used. Can the group of two voters favorably impact the results through insincere voting?
SOLN: K currently wins. Yes. For example, by exchanging L and M, L will win instead.

e. Suppose the Borda Count method is used. Suppose the group of six suspect that the group of two intends to insincerely exchange M and L in their rankings. Can the group of six counteract in order to protect K as the winner?
SOLN: Yes. If the group of two exchange M and L and the group of six exchange L and N, K will still win.

5. Consider an 11-member committee that must choose one of three alternatives: X, Y, or Z, using the Hare system. Their schedule of preferences is shown below. Who wins?

	Number of Voters		
	5	4	2
First choice	Z	X	Y
Second choice	Y	Y	X
Third choice	X	Z	Z

a. Is it possible for the group of five voters to change the outcome in a way that would benefit them?
SOLN: X wins. If the group of five voters exchange their rankings of Y and Z, then Y wins.

b. Is it possible for the group of two voters to change the outcome in a way that would benefit them?
SOLN: The group of two voters cannot change the outcome by insincerely changing their preference ordering.

c. However, the committee suspects that the group of five plans to insincerely reorder their preferences as Y, Z, X. How can the group of four respond?
SOLN: If the group of five exchange Y and Z, then Y wins. The group of four cannot retaliate against this change.

6. A group of 22 young people must decide whether to go to the beach (B), the mountain (M), or the zoo (Z) on a field trip. Their preference rankings are summarized in the table below, and the decision will be made using a Borda count. Who wins the vote?

	Number of Voters		
	10	8	4
First choice	B	M	Z
Second choice	M	B	M
Third choice	Z	Z	B

a. Can the four voters in the last column change the results of the vote to their favor by changing their preference rankings?
SOLN: M wins. No, they cannot change the results through insincere voting

b. Can the 10 voters in the first column change the results of the vote to their favor by changing their preference rankings?

SOLN: Yes; by exchanging M and Z in their rankings, B would win instead.

7. Consider the following preference table:

	Number of voters			
	4	6	8	4
First choice	D	C	A	B
Second choice	C	B	D	A
Third choice	B	D	C	C
Fourth choice	A	A	B	D

a. Who wins using plurality? Could the four voters who most prefer B vote insincerely to change the outcome in a way that would benefit them?

SOLN: A wins. If the group of four insincerely votes for C instead of D, then C would win.

b. Could the six voters who most prefer C vote insincerely to change the outcome in a way that would benefit them?

SOLN: No. They cannot force B to win through insincere voting, and A was their second choice.