Math 54 – Chapters 7 and 8 Test – Spring '10   Name_____

Show all work for credit. Write all responses on separate paper.

1. Simplify the following as completely as possible:

   a. $\dfrac{\left(-x^3\right)^4}{-\left(x^6\right)^2}$

   b. $\dfrac{-2^3-3^2}{-5^2-9}$

   c. $\dfrac{\left(2-3\right)^{13}-2^5}{-6^2+3}$

   d. $\dfrac{2\left(-4\right)^3+4\left(3\right)^2}{\left(-4+3\right)^3}$

2. Simplify the following as completely as possible. The final answer should be expressed with positive exponents only. (Assume all the variable represent positive quanitities.)

   a. $\left(\dfrac{a^{-2}b^{-5}}{7a^{-5}b^{-3}}\right)^{-2}$

   b. $\dfrac{\left(4m^{-2}n^{-1}\right)^{-1}}{\left(m^{-2}n^{-1}\right)^{-2}}$

3. Consider the expression $\dfrac{1}{x^{-1}+y^{-1}}$

   a. Simplify the expression (that is, write an equivalent expression without negative exponents.)

   b. Evaluate the expression for $x = 3$ and $y = -4$.

4. If the mass of one atom of iron is 9.3×10^{-23} gram, how many atoms are there in 8 grams of iron?

5. Evaluate the polynomial $-3x^2+11x-5$ for $x = \dfrac{2}{3}$.

6. The concentration C of a medication in the blood stream (in milligrams per millimeter) m minutes after the medication is taken is described by the formula $C = 10 + 20m - 0.2m^2$. Use this formula to find the concentration of the medication in the bloodstream after 20 minutes.

7. Expand and combine like terms to write each as a polynomial in descending powers:

   a. $\left(x^4 - 3x^2 + 3\right)\left(x^4 + 3x^2 + 3\right)$

   b. $\left(2x-1\right)^2 - \left(x+2\right)^2$

8. The width of a rectangle is 6 less than 3 times its length. Express each in terms of the length, L:

   a. The perimeter of the rectangle

   b. The area of the rectangle

9. Factor each as completely as possible:

   a. $6x^2yz^3 + 54xyz^2 - 24x^2y^2z^2$

   b. $m^2 + mn + 9m + 9n$

10. Factor the special product as completely as possible:

   a. $4x^2 - y^2$

   b. $4x^2 + 12x + 9$

11. Solve the equation. Use the factoring method with the zero product principle.

   a. $2x^2 = 11x - 12$

   b. $\left(x-4\right)^2 = 2x^2 - 11x - 12$

12. Divide to find the quotient and remainder: $\dfrac{x^3 + 2x^2 + 144}{x+6}$

Math 54 – Chapters 7 and 8 Test Solutions – Spring '10   .

1. Simplify the following as completely as possible:

   a. $\dfrac{\left(-x^3\right)^4}{-\left(x^6\right)^2} = \dfrac{x^{12}}{-x^{12}} = -1$

   b. $\dfrac{-2^3 - 3^2}{-5^2 - 9} = \dfrac{-8 - 9}{-25 - 9} = \dfrac{-17}{-34} = \dfrac{1}{2}$

   c. $\dfrac{(2-3)^{13} - 2^5}{-6^2 + 3} = \dfrac{(-1)^{13} - 32}{-36 + 3} = \dfrac{-1 - 32}{-33} = \dfrac{-33}{-33} = 1$

   d. $\dfrac{2(-4)^3 + 4(3)^2}{(-4+3)^3} = \dfrac{2(-64) + 4(9)}{(-1)^3} = \dfrac{-128 + 36}{-1} = 92$

2. Simplify the following as completely as possible. The final answer should be expressed with positive exponents only. (Assume all the variable represent positive quanitities.)

   a. $\left(\dfrac{a^{-2}b^{-5}}{7a^{-5}b^{-3}}\right)^{-2} = \left(\dfrac{a^3}{7b^2}\right)^{-2} = \dfrac{49b^4}{a^6}$

   b. $\dfrac{\left(4m^{-2}n^{-1}\right)^{-1}}{\left(m^{-2}n^{-1}\right)^{-2}} = \dfrac{m^2 n}{4m^4 n^2} = \dfrac{1}{4m^2 n}$

3. Consider the expression $\dfrac{1}{x^{-1} + y^{-1}}$

   a. Simplify the expression (that is, write an equivalent expression without negative exponents.)

   SOLN: $\dfrac{1}{x^{-1} + y^{-1}} = \dfrac{1}{\dfrac{1}{x} + \dfrac{1}{y}} = \dfrac{1}{\dfrac{y}{xy} + \dfrac{x}{xy}} = \dfrac{1}{\dfrac{y+x}{xy}} = \dfrac{xy}{y+x}$

   b. Evaluate the expression for $x = 3$ and $y = -4$.

   SOLN: $\dfrac{3(-4)}{-4+3} = \dfrac{-12}{-1} = 12$

4. If the mass of one atom of iron is 9.3×10^{-23} gram, how many atoms are there in 8 grams of iron?

   SOLN: $\dfrac{8\,\text{grams}}{9.3 \times 10^{-23}\,\text{grams/atom}} = \dfrac{8}{9.3 \times 10^{-23}}\,\text{atoms} = 0.86 \times 10^{23}\,\text{atoms} = 8.6 \times 10^{22}\,\text{atoms}$

5. Evaluate the polynomial $-3x^2 + 11x - 5$ for $x = \dfrac{2}{3}$.

   SOLN: $-3\left(\dfrac{2}{3}\right)^2 + 11\left(\dfrac{2}{3}\right) - 5 = -3\left(\dfrac{4}{9}\right) + \dfrac{22}{3} - \dfrac{15}{3} = \dfrac{-4 + 22 - 15}{3} = 1$

6. The concentration C of a medication in the blood stream (in milligrams per millimeter) m minutes after the medication is taken is described by the formula $C = 10 + 20m - 0.2m^2$. Use this formula to find the concentration of the medication in the bloodstream after 20 minutes.

   SOLN: $C = 10 + 20(20) - 0.2(20)^2 = 10 + 400 - 0.2(400) = 410 - 80 = 330$ mg/mL.

7. Expand and combine like terms to write each as a polynomial in descending powers.

a.
$$\left(x^4-3x^2+3\right)\left(x^4+3x^2+3\right)=x^4\left(x^4+3x^2+3\right)-3x^2\left(x^4+3x^2+3\right)+3\left(x^4+3x^2+3\right)$$
$$=x^8+3x^6+3x^4-3x^6-9x^4-9x^2+3x^4+9x^2+9=\boxed{x^8-3x^4+9}$$

b. $\left(2x-1\right)^2-\left(x+2\right)^2=4x^2-4x+1-\left(x^2+4x+4\right)=3x^2-8x-3$

8. The width of a rectangle is 6 less than 3 times its length. Express each in terms of the length, L:

a. The perimeter of the rectangle
SOLN: Perimeter $= 2W + 2L =$
$2(3L - 6) + 2L = 8L - 12$

b. The area of the rectangle
SOLN: area $= LW = L(3L - 6)$

9. Factor each as completely as possible:

a. $6x^2yz^3+54xyz^2-24x^2y^2z^2$
SOLN: $= 6xyz^2\left(xz+9-4xy\right)$

b. $m^2+mn+9m+9n$
SOLN:
$m\left(m+n\right)+9\left(m+n\right)=\left(m+9\right)\left(m+n\right)$

10. Factor the special product as completely as possible:

a. $4x^2-y^2=\left(2x-y\right)\left(2x+y\right)$

b. $4x^2+12x+9=\left(2x+3\right)^2$

11. Solve the equation. Use the factoring method with the zero product principle.

a.
$2x^2=11x-12$
$\Leftrightarrow 2x^2-11x+12=0$
$\Leftrightarrow 2x^2-8x-3x+12=0$
$\Leftrightarrow 2x\left(x-4\right)-3\left(x-4\right)=0$
$\Leftrightarrow \left(2x-3\right)\left(x-4\right)=0$
So $x = 3/2$ or $x = 4$.

b.
$\left(x-4\right)^2=2x^2-11x-12$
$\Leftrightarrow x^2-8x+16=2x^2-11x-12$
$\Leftrightarrow 0=x^2-3x-28$
$\Leftrightarrow 0=\left(x-7\right)\left(x+4\right)$
So $x = 7$ or $x = -4$

12. Divide to find the quotient and remainder: $\dfrac{x^3+2x^2+144}{x+6}$

$$
\begin{array}{r}
x^2 - 4x + 24 \\
x + 6 \,\overline{\smash{\big)}\, x^3 + 2x^2 + 0 + 144} \\
\underline{x^3 + 6x^2} \\
-4x^2 + 0 \\
\underline{-4x^2 - 24x} \\
24x+144 \\
\underline{24x+144} \\
0
\end{array}
$$