

Name \_\_\_\_\_

Instructions: Write all your responses to the following on separate paper. Show your work for credit. Take as much space as you need. Do not crowd into corners. Do not use an electronic calculator.

- Simplify the expression as a single ratio:  $\frac{x}{x+3} - \frac{4}{x^2-9}$
- Solve the following equation. Be sure to check each answer in the original equation if you multiply both sides by an expression that contains the variable.  $\frac{6}{x-1} + \frac{5}{x+1} = \frac{45}{x^2-1}$
- Solve the following equation. Be sure to check each answer in the original equation if you multiply both sides by an expression that contains the variable.  $\frac{2x}{x-1} + \frac{3}{x+1} = \frac{15}{x^2-1}$
- One train travels 10 miles per hour faster than another. The slower train goes 80 miles in same time it takes the faster to go 100 miles. What are their speeds? Use algebra to solve the problem.
- A motor boat travels 20 mph in still water. A boat takes 4 hours longer to motor 60 miles upstream than it takes to motor 60 miles downstream. What is the speed of the stream? Use algebra to solve the problem.
- Simplify the complex fraction.  $\frac{2 + \frac{6}{x+1}}{2 + \frac{3}{x+1}}$ .  
Hint: As a check note that when  $x = 0$  this fraction is  $\frac{6}{5}$  and when  $x = 2$  the fraction is  $\frac{4}{3}$ .
- Solve the following proportion:  $\frac{10}{x} = \frac{x-7}{6}$ .
- Simplify the following roots. If the value is not a real number, write "not real."  
a.  $\sqrt{-100}$                       b.  $-\sqrt{1}$                       c.  $\sqrt[3]{-216}$
- Simplify:  
a.  $\sqrt{24} + \sqrt{96}$                       b.  $\sqrt{\frac{16}{27}}$                       c.  $\frac{\sqrt{32x^5}}{\sqrt{9x^2}}$
- Consider the equation,  $y = \frac{15}{x}$   
a. Make a table of values for the equation including the points where  $x = -15, -5, -3, -1, 1, 3, 5$  and  $15$ .  
b. Make a plot of these points and draw a smooth curve connecting them.  
c. Find an equation for the line intersecting this curve where  $x = -5$  and where  $x = 3$ .

## Math 54 – Beginning Algebra – Chapters 7.1 – 8.2 & 6 Test Solutions – Spring '11

1. Simplify the expression as a single ratio  $\frac{x}{x+3} - \frac{4}{x^2-9}$

$$\text{SOLN: } \left(\frac{x}{x+3}\right)\left(\frac{x-3}{x-3}\right) - \frac{4}{x^2-9} = \frac{x(x-3)-4}{x^2-9} = \frac{x^2-3x-4}{x^2-9} = \frac{(x-4)(x+1)}{(x-3)(x+3)} = 1 - \frac{3x-5}{x^2-9}$$

2. Solve the following equation. Be sure to check each answer in the original equation if you multiply both sides by an expression that contains the variable.  $\frac{6}{x-1} + \frac{5}{x+1} = \frac{45}{x^2-1}$

$$\text{SOLN: } \frac{6}{x-1} + \frac{5}{x+1} = \frac{45}{x^2-1} \Leftrightarrow 6(x+1) + 5(x-1) = 45 \Leftrightarrow 11x+1 = 45 \Leftrightarrow 11x = 44 \Leftrightarrow \boxed{x=4}$$

To be sure,  $\frac{6}{3} + \frac{5}{5} = \frac{45}{15}$  is true.

3. Solve the following equation. Be sure to check each answer in the original equation if you multiply both sides by an expression that contains the variable.  $\frac{2x}{x-1} + \frac{3}{x+1} = \frac{15}{x^2-1}$

$$\text{SOLN: } 2x(x+1) + 3(x-1) = 15 \Leftrightarrow 2x^2 + 5x - 18 = 0 \Leftrightarrow (2x+9)(x-2) = 0 \Leftrightarrow \boxed{x=2 \text{ or } x=-\frac{9}{2}}$$

4. One train travels 10 miles per hour faster than another. The slower train goes 80 miles in same time it takes the faster to go 100 miles. What are their speeds? Use algebra to solve the problem.

SOLN: Let  $x$  = the speed of the slower train. Then the faster train's speed is  $x + 10$ . An equation that says the times are the same is  $\frac{80}{x} = \frac{100}{x+10} \Leftrightarrow 80x + 800 = 100x \Leftrightarrow x = 40$ . So the slower train is traveling at 40mph and the faster train is traveling at 50mph.

5. A motor boat travels 20 mph in still water. A boat takes 4 hours longer to motor 60 miles upstream than it takes to motor 60 miles downstream. What is the speed of the stream? Use algebra to solve the problem.

SOLN: Let  $x$  = the speed of the stream. Then the speed of the boat upstream is  $20 - x$  and the speed downstream is  $20 + x$  and the time going upstream is 4 hours more than the time going downstream so

$$\frac{60}{20-x} = \frac{60}{20+x} + 4 \Leftrightarrow 60(20+x) = 60(20-x) + 4(400 - x^2) \Leftrightarrow 1200 + 60x = 1200 - 60x + 1600 - 4x^2$$

$$4x^2 + 120x - 1600 = 0 \Leftrightarrow x^2 + 30x - 400 = 0 \Leftrightarrow (x-10)(x+40) = 0 \text{ Since } x > 0, \text{ the only sensible}$$

solution is  $x = 10$ mph. As a check  $\frac{60}{10} = \frac{60}{30} + 4$ , is true. So the speed of the stream is 10 mph.

6. Simplify the complex fraction.  $\frac{2 + \frac{6}{x+1}}{2 + \frac{3}{x+1}}$ .

Hint: As a check note that when  $x = 0$  this fraction is  $\frac{8}{5}$  and when  $x = 2$  the fraction is  $\frac{4}{3}$ .

SOLN: Multiply the numerator and denominator by  $x + 1$  to obtain  $\frac{2(x+1)+6}{2(x+1)+3} = \frac{2x+8}{2x+5}$

7. Solve the following proportion:  $\frac{10}{x} = \frac{x-7}{6}$ .

SOLN:

$$6x\left(\frac{10}{x}\right) = 6x\left(\frac{x-7}{6}\right) \Leftrightarrow 60 = x^2 - 7x \Leftrightarrow x^2 - 7x - 60 = 0 \Leftrightarrow (x-12)(x+5) = 0 \Leftrightarrow \boxed{x=12 \text{ or } x=-5}$$

8. Simplify the following roots. If the value is not a real number, write "not real."

a.  $\sqrt{-100}$  is not a real number    b.  $-\sqrt{1} = -1$     c.  $\sqrt[3]{-216} = -6$

9. Simplify:

a.  $\sqrt{24} + \sqrt{96} = 2\sqrt{6} + 4\sqrt{6} = 6\sqrt{6}$     b.  $\sqrt{\frac{16}{27}} = \frac{\sqrt{16}}{\sqrt{27}} = \frac{4}{\sqrt{9}\sqrt{3}} = \frac{4}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{9}$     c.  $\frac{\sqrt{32x^5}}{\sqrt{9x^2}} = \frac{4x\sqrt{2x}}{3}$

10. Consider the equation,  $y = \frac{15}{x}$

- Make a table of values for the equation including the points where  $x = -15, -5, -3, -1, 1, 3, 5$  and  $15$ .
- Make a plot of these points and draw a smooth curve connecting them.
- Find an equation for the line intersecting this curve where  $x = -5$  and where  $x = 3$ .

SOLN The slope of the line is  $m = \frac{5 - (-3)}{3 - (-5)} = \frac{8}{8} = 1$  so an equation is  $y - 5 = 1(x - 3) \Leftrightarrow y = x + 2$

$x$	-15	-5	-3	-1	1	3	5	15
$y$	-1	-3	-5	-15	15	5	3	1

