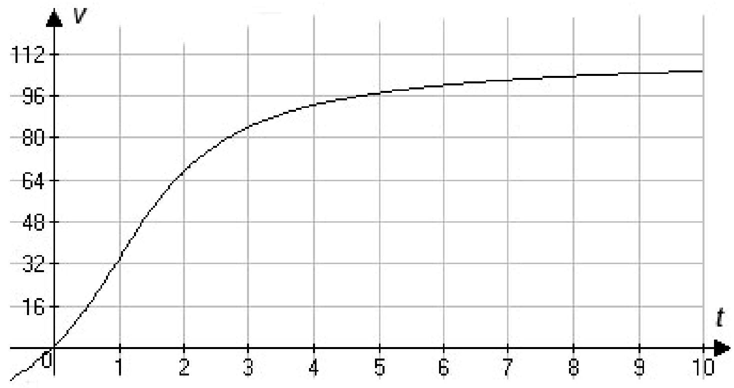
Math 1B – Sections §5.1 - §5.6 Test – Fall 2008 Name__________________

Instructions: Write all responses on separate paper. Show all work for credit. Do not use a calculator.

1. The velocity graph of an experimental rocket accelerating from rest to a velocity of about 104 km/h over a period of 10 seconds is shown. Approximate (to the nearest hundredth of a km) the distance travelled by the rocket over this time using $n = 2$ evenly spaced subintervals and
   a. right endpoints as sample points
   b. the trapezoidal method
   c. the midpoint method



2. Consider the Riemann sums for $\int_2^4 \sin(\pi x)dx$ with n evenly space intervals.
   a. Write the length of each subinterval in terms of n.
   b. Write the right endpoint of the kth subinterval in terms of n and k.
   c. Write the integral as the limit of Riemann sums in terms of n and k. No x.

3. Evaluate the integral by interpreting it in terms of the areas of well known figures such as rectangles, triangles or circles. Draw a diagram for each.
   a. $\int_2^3 \left(\frac{x}{2}+1\right)dx$
   b. $\int_0^3 \sqrt{9-x^2}\,dx$
   c. $\int_0^3 3-\sqrt{9-x^2}\,dx$

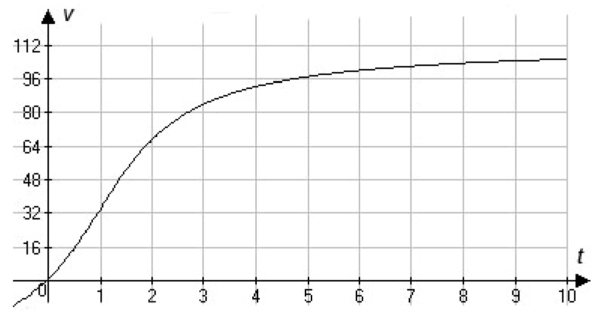
4. Use the Fundamental Theorem of Calculus to find the derivative of the given function.
   a. $g(x) = \int_1^x \frac{1}{v}dv$
   b. $H(z) = \int_z^{z^2} \sin(x^2)dx$ *Hint*: Use the property $\int_a^b f dt = \int_a^c f dt + \int_c^b f dt$

5. Can you find a non-zero solution to the equation $\int_0^x \sin(t^2)dt = x$? Discuss why or why not.

6. Evaluate the definite integral
   a. $\int_0^{\pi/8} \sec^2(2x)\,dx$
   b. $\int_0^1 \sqrt{3-3x}\,dx$
   c. $\int_0^4 \frac{x}{\sqrt{4x+9}}dx$
   d. $\int_0^{\ln 2} t^2 e^t\,dt$
   e. $\int_0^t \sin(t-s)\,ds$
   f. $\int_0^{\pi} e^x \cos(4x)\,dx$

Math 1B – Sections §5.1 - §5.6 Test Solutions – Fall 2008

1. The velocity graph of an experimental rocket accelerating from rest to a velocity of about 104 km/h over a period of 10 seconds is shown. Approximate (to the nearest hundredth of a km) the distance travelled by the rocket over this time using $n = 2$ evenly spaced subintervals and



   a. right endpoints as sample points
      SOLN: 5*(98+106)/3600 = 204/720 = 17/60 ~ 0.28 km
   b. the trapezoidal method
      SOLN: $\frac{\Delta x}{2}\left(f(0)+2f(5)+f(10)\right) \approx 2.5\left(0+2*98+106\right) \approx 0.21$ km
   c. the midpoint method
      SOLN: $\Delta x\left(f(2.5)+f(7.5)\right) \approx 5(78+102)/3600 = 0.25$

2. Consider the Riemann sums for $\int_2^4 \sin(\pi x)dx$ with n evenly space intervals.
   a. Write the length of each subinterval in terms of n.
      SOLN: $\Delta x = \frac{b-a}{n} = \frac{4-2}{n} = \frac{2}{n}$
   b. Write the right endpoint of the kth subinterval in terms of n and k.
      SOLN: $x_k = a + k\Delta x = 2 + \frac{2k}{n}$
   c. Write the integral as the limit of Riemann sums in terms of n and k. No x.|
      SOLN: $\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(\pi\left(2+\frac{2k}{n}\right)\right)\frac{2}{n} = \lim_{n\to\infty}\frac{2}{n}\sum_{k=1}^{n}\sin\left(\frac{2k\pi}{n}\right)$

3. Evaluate the integral by interpreting it in terms of the areas of well known figures such as rectangles, triangles or circles. Draw a diagram for each.
   a. $\int_2^3\left(\frac{x}{2}+1\right)dx$ is the area of a trapezoid with base = 1 and average height = $\frac{2+2.5}{2} = 2.25$ so the area is 2.25
   b. $\int_0^3\sqrt{9-x^2}\,dx$ is the area of a quarter circle with radius 3, this is $\frac{9\pi}{4}$
   c. $\int_0^3 3-\sqrt{9-x^2}\,dx$ is what's left when the quarter circle's area (above) is subtracted from the area of a 3 by 3 square, or $9-\frac{9\pi}{4} \approx \frac{36-29.3}{4} \approx 1.7$

4. Use the Fundamental Theorem of Calculus to find the derivative of the given function.
   a. $g(x) = \int_1^x \frac{1}{v}dv \Rightarrow g'(x) = \frac{1}{x}$ "That was easy!"

$$H(z)=\int_z^{z^2}\sin\left(x^2\right)dx \Rightarrow H'(z)=\frac{d}{dz}\int_z^0\sin\left(x^2\right)dx+\int_0^{z^2}\sin\left(x^2\right)dx$$

b. $$=-\frac{d}{dz}\int_0^z\sin\left(x^2\right)dx+2z\frac{d}{du}\int_0^u\sin\left(x^2\right)dx$$
$$=-\sin\left(z^2\right)+2z\sin\left(z^4\right)$$

5. Can you find a non-zero solution to the equation $\int_0^x \sin\left(t^2\right)dt = x$? Discuss why or why not.

SOLN: Note that $x=\int_0^x 1dt$ and that $\sin\left(t^2\right)\le 1$ for all t. Thus the right side is accumulating area at a rate of 1 unit per unit x while the left side accumulating area at a rate less than this most of the time, and since it starts out accumulating area at a slower rate, there's no way the areas can be the same.

6. Evaluate the definite integral

a. $\int_0^{\pi}\sec^2(2x)dx$ exists only if $\int_0^{\pi/4}\sec^2(2x)dx=\lim_{b\to\pi/4^-}\int_0^b\sec^2(2x)dx$ exists, and $\lim_{b\to\pi/4^-}\int_0^b\sec^2(2x)dx=\lim_{b\to\pi/4^-}\frac{1}{2}\tan(2x)\Big|_0^b=\lim_{b\to\pi/4^-}\frac{1}{2}\tan(2b)=\infty$ does not exist.

However, $\int_0^{\pi/8}\sec^2(2x)dx=\frac{1}{2}\tan(2x)\Big|_0^{\pi/8}=\frac{1}{2}$

b. Substitute $u = 3 - 3x$ so that $du = -3dx$

$$\int_0^1\sqrt{3-3x}\,dx=\int_3^0\sqrt{u}\left(-\frac{1}{3}du\right)=\frac{1}{3}\int_0^3 u^{1/2}du=\frac{2}{9}u^{3/2}\Big|_0^3=\frac{2\sqrt{3}}{3}$$

c. Substitute $u=4x+9 \Rightarrow du=4dx$ and $x=\frac{u-9}{4}$ to get

$$\int_0^4\frac{x}{\sqrt{4x+9}}dx=\int_9^{25}\frac{u-9}{4\sqrt{u}}\left(\frac{du}{4}\right)=\frac{1}{16}\int_9^{25}u^{1/2}-9u^{-1/2}du=\frac{1}{16}\left(\frac{2u^{3/2}}{3}-18u^{1/2}\right)\Bigg|_9^{25}$$
$$=\frac{1}{16}\left[\left(\frac{250}{3}-90\right)-(18-54)\right]=\frac{1}{16}\left(36-\frac{20}{3}\right)=\frac{11}{6}$$

d. Two integrations by parts are required here:

$\int_0^{\ln 2}t^2e^t dt$ with $\boxed{\begin{array}{ll} u=t^2 & dv=e^t dt \\ du=2tdt & v=e^t\end{array}}$

$=t^2e^t\Big|_0^{\ln 2}-2\int_0^{\ln 2}te^t dt$ with $\boxed{\begin{array}{ll} u=t & dv=e^t dt \\ du=dt & v=e^t\end{array}}$

$$=2(\ln 2)^2-2\left[te^t\Big|_0^{\ln 2}-\int_0^{\ln 2}e^t dt\right]=2(\ln 2)^2-4\ln 2+2$$

e. Let $u = t - s$ so that $du = -ds$ and $\int_0^t\sin(t-s)ds=\int_t^0\sin u(-du)=\int_0^t\sin u\,du=1-\cos t$

f. Do parts with $\boxed{\begin{array}{ll} u = e^x & dv = \cos(4x)\,dx \\ du = e^x dx & v = \frac{1}{4}\sin(4x) \end{array}}$ so that

$I = \int_0^{\pi} e^x \cos(4x)\,dx = \frac{e^x}{4}\sin(4x)\Big|_0^{\pi} - \frac{1}{4}\int_0^{\pi} e^x \cos(4x)\,dx$ and doing parts again with

$\boxed{\begin{array}{ll} u = e^x & dv = -\sin(4x)\,dx \\ du = e^x dx & v = \frac{1}{4}\cos(4x) \end{array}}$ yields $I = \frac{e^x}{16}\cos(4x)\Big|_0^{\pi} - \frac{1}{16}I \Leftrightarrow I = \frac{e^{\pi}-1}{17}$