Math 1B—Calculus II – Fair Game for Chapters 8 and 10 Problems

1. Find the length of the curve $y = \frac{1}{9}\left(x^2 + 6\right)^{3/2}$, $0 \le x \le 1$.

2. Find the length of the curve $y = \int_0^x \sqrt{\cos^2 t - 1}\, dt$, $0 \le x \le \pi$.

3. Find the surface area generated by rotating about the y–axis the curve $x = \sin(y)$ for $0 \le y \le \pi$.

4. Find the surface area of surface generated by rotating the curve $y = x^{-3}$ about the x-axis for $x \ge 1$.

5. A gate in an irrigation canal is constructed in the form of a semicircle with diameter 5 meters at the top. If the canal is filled to a depth of 3 meters, find the hydrostatic force on one side of the gate.

6. Find the centroid of the region bounded by $y = \sin(x)$ and $y = 2\sin(x)$, $0 \le x \le \pi$.

7. Find the moments and center of mass of the system of objects that have masses 4, 7 and 11 at the points (–2,3), (1,1) and (3,–2), respectively.

8. Lengths of human pregnancies are normally distributed with mean 268 days and standard deviation 15 days. What percentage of pregnancies last between 250 days and 280 days? Hint: use the normal probability density function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$

9. Let $f(x) = kx\left(1 - x^2\right)$ if $0 \le x \le 1$ and $f(x) = 0$ if $x < 0$ or $x > 1$.
   a. For what value of k is $f(x)$ a probability density function?
   b. For that value of k, find $P(X \ge 0)$
   c. Find the mean.

10.

11. Find the area in the xy–plane enclosed by the curve described by each pair of parametric equations. In each case, include a table of (t, x, y) values, as needed, to analyze the graph:
    a. $x = 1 + 3\cos(\pi t)$ and $y = 3 - 2\sin(\pi t)$.

t	x	y

b. $x = \sin(2t)$ and $y = \cos(t)$

t	x	y

12. Find the area in the *xy*–plane enclosed by the *x*-axis and the curve described by the parametric equations $x = 1 + e^t$ and $y = 3t - t^2$.

13. Consider the parametric equations describing a hyperbola: $\boxed{\begin{aligned} x &= 1 + 2\sec t \\ y &= 3 + 4\tan t \end{aligned}}$

    a. Write the equation for the hyperbola in standard form by specifying values for *a*, *b*, *h*, and *k* in the formula $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b} = 1$

    Hint: Recall the identity $\sec^2 t - \tan^2 t = 1$

    b. Find a value of *t* so that the tangent line at (*x*(*t*), *y*(*t*)) has slope = 4.

    c. Find the value of $\left|\dfrac{d^2y}{dx^2}\right|$ where $x = 5$.

14. Find the area of the loop formed by $\begin{aligned} x &= t^2 + 2 \\ y &= t\left(t^2 - 9\right) \end{aligned}$

    Hint: The loop closes where the curve intersects itself: where we can find two different parameter values, $t_1 \neq t_2$, such that $x(t_1) = x(t_2)$ and $y(t_1) = y(t_2)$.

15. Find the area that lies inside the curve $r = 2 + \cos\vartheta$ and outside $r = \cos(2\vartheta)$.

16. The area of the surface generated by rotating the polar curve $r = f(\theta)$ for $a \le \theta \le b$ about the polar axis (the *x*-axis) is $S = \int_a^b 2\pi r \sin\theta \sqrt{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}\, d\theta$. Use this formula to find the surface area generated by rotating the lemniscate $r^2 = \cos 2\vartheta$ about the polar axis.