Math 1B – Calculus – Fair Game for Chapter 7 Test – Fall '11

1. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

   a. $\int_1^\infty \frac{1}{(2x-1)^2}\,dx$ b. $\int_0^\infty xe^{-x^2}\,dx$ c. $\int_{-\infty}^\infty \frac{x^4}{64+x^6}\,dx$

   d. $\int_{-\infty}^\infty \frac{x^4}{64+x^2}\,dx$ e. $\int_0^\infty \frac{x^4}{4+x^2}\,dx$ f. $\int_0^\infty \frac{x^3}{4+x^2}\,dx$

   g. $\int_0^\infty \frac{1}{x^2\sqrt{x^2+1}}\,dx$ h. $\int_1^\infty \frac{1}{x^2\sqrt{x^2-1}}\,dx$ i. $\int_1^\infty \frac{1}{(x-1)^2\sqrt{x}}\,dx$

2. Use the comparison theorem to show that the integral is either convergent or divergent.

   a. $\int_{100}^\infty \frac{1-e^{-x}}{x^2}\,dx$ b. $\int_{100}^\infty \frac{1-e^{-x}}{x}\,dx$ c. $\int_3^\infty \frac{\sin^2 x}{\sqrt{x^3}-3}\,dx$

3. Consider the integral $\int_0^\infty x^n e^{-x}\,dx$.

   a. Evaluate the integral for $n = 0, 1, 2,$ and 3.
   b. Make a guess about the value of the integral in terms of n.
   c. Prove your guess is right by using mathematical induction.

4. If $f(t)$ is continuous for $t \geq 0$, the Laplace transform of f is the function F defined by

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

   and the domain of F is the set consisting of all numbers s for which the integral converges.
   Find the Laplace transforms of the following functions:
   a. $f(t) = 2$
   b. $f(t) = t^2$

5. Determine the value of a so that $\int_0^a \frac{t}{t^2+1}\,dt = e^{1000}$

6. If the integrals $\int_a^\infty f(t)\,dt$ and $\int_{-\infty}^a f(t)\,dt$ are convergent then it's possible to define $\int_{-\infty}^\infty f(t)\,dt = \int_{-\infty}^a f(t)\,dt + \int_a^\infty f(t)\,dt$. With this in mind, is possible to define $\int_{-\infty}^\infty \frac{t}{t^2+1}\,dt$?

7. Find the value of the constant C for which the integral $\int_0^\infty \frac{t}{t^2+1} - \frac{C}{3t+1}\,dt$ and evaluate the integral.

8. Show that if $a > -1$ and $b > a + 1$, then the integral $\int_0^\infty \frac{y^a}{y^b+1}\,dy$ is convergent.