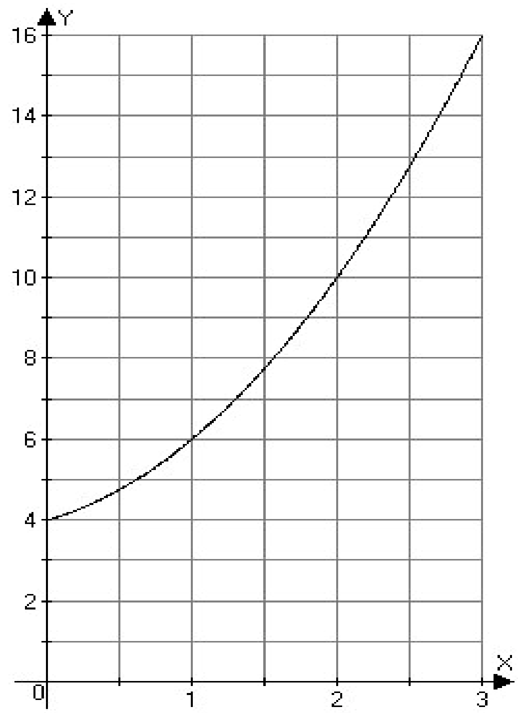
Math 1B - Chapter 5 test Name___________________________

Write all responses on separate paper. Show work for credit. No "graphing" calculator.

1. Consider the function whose graph is shown at right.



 a. Use the graph to approximate $\int_0^3 f(x)\,dx \approx R_3$, the Riemann sum formed with 3 subintervals of equal length and using right endpoints as sample points.

 b. Use the graph to approximate $\int_0^3 f(x)\,dx \approx L_3$, the Riemann sum formed with 3 subintervals of equal length and using left endpoints as sample points.

 c. Use the graph to approximate $\int_0^3 f(x)\,dx \approx M_3$, the Riemann sum formed with 3 subintervals of equal length and using midpoints as sample points (try to approximate to the nearest tenth.)

 d. Put these in order from smallest to largest: R_3, L_3, $(R_3 + L_3)/2$ and M_3.

2. Determine a region whose area is determined by the given limit. Do not evaluate the limit.

 a. $\lim_{n\to\infty} \frac{9}{n}\sum_{i=1}^{n} \sin\left(\frac{9\pi i}{n}\right)$ b. $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \left(4+\frac{2i}{n}\right)^3$

3. Use the definition of the integral to evaluate the integral $\int_0^4 \left(x+3x^3\right)dx$. Note: you can use the Evaluation Theorem to check your work, but you *must* use the definition for credit.

4. Evaluate the integral by interpreting it in terms of familiar area formulae.

 a. $\int_0^1 \left(1+\sqrt{1-x^2}\right)dx$ b. $\int_{-2}^1 \left(2-|x|\right)dx$

5. Use Part 1 or Part 2 of the Fundamental theorem of Calculus to evaluate the expression:

 a. $\frac{d}{dx}\int_0^{\pi} \cos\left(t^2\right)dt$ b. $\int_0^{\pi} \frac{d}{dt}\sin\left(t^2\right)dt$ c. $\frac{d}{dx}\int_{x^2}^{x^3} \left(t^2\right)dt$

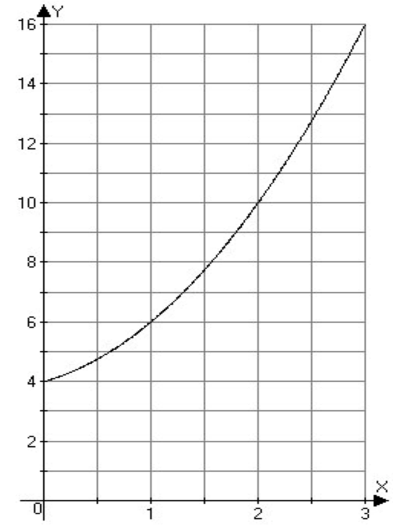
6. Use substitution to simplify the following definite integrals. For each, explicitly state what the components of your substitution are (on separate paper):

 a. $\int_0^{\pi/4} \tan^3\theta \sec^2\theta \, d\theta$ $u = \tan\theta$ $du =$ ____________?

 b. $\int_0^1 x^2\left(x^3+1\right)^8 dx$ $u =$ _____ ? $du =$ ___________?

Math 1B - Chapter 5 test Solutions – Fall ‘11

1. Consider the function whose graph is shown at right.



   a. Use the graph to approximate $\int_0^3 f(x)\,dx \approx R_3$

   SOLN: R_3 = 1*(6+10+16) = 32

   b. Use the graph to approximate $\int_0^3 f(x)\,dx \approx L_3$

   SOLN: L_3 = 1*(4+6+10) = 20

   c. Use the graph to approximate $\int_0^3 f(x)\,dx \approx M_3$

   (try to approximate to the nearest tenth.)
   SOLN: M_3 = 1*(4.8+7.8+12.8) = 25.4

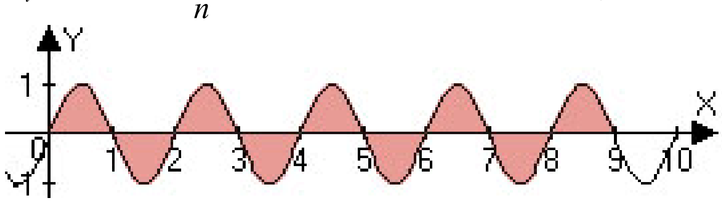
   d. Put these in order from smallest to largest:
   R_3, L_3, $(R_3 + L_3)/2$ and M_3.
   SOLN: $L_3 < M_3 < (R_3 + L_3)/2 < R_3$,

2. Determine a region whose area is determined by the given limit. Do not evaluate the limit.

   a. $\lim_{n\to\infty} \frac{9}{n}\sum_{i=1}^{n} \sin\left(\frac{9\pi i}{n}\right)$ SOLN: From $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$ we conjecture that

   $\Delta x = \frac{b-a}{n} = \frac{9}{n}$. This fist if a = 0, b = 9 and the integrand function is sin(πx) and so

   $x_i = a + i\Delta x = \frac{9i}{n}$. Thus the area represented by limit is $\int_0^9 \sin(\pi x)\,dx$

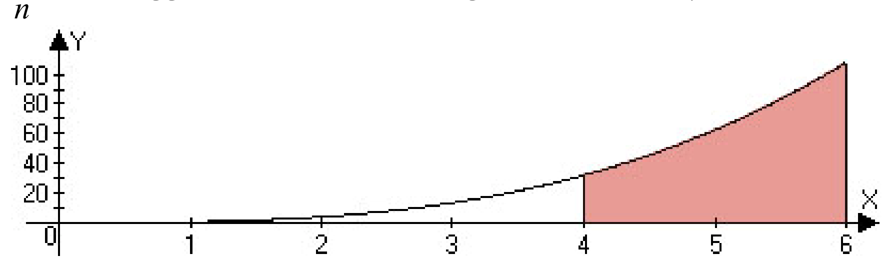


   b. $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \left(4 + \frac{2i}{n}\right)^3$ SOLN: Here it is reasonable to suppose that the integrand function

   is $y = x^3$ and so the sample point is $x_i = a + i\Delta x = 4 + \frac{2i}{n}$ which naturally suggests

   that a = 4 and $\Delta x = \frac{2}{n}$. This suggests we revise our integrand function to $y = x^3/2$ and so

   the area is $\int_4^6 \frac{x^3}{2}\,dx$



3. Use the definition of the integral to evaluate the integral $\int_0^4 \left(x + 3x^3\right)dx$. Note: you can use the Evaluation Theorem to check your work, but you *must* use the definition for credit.
   SOLN:

$$\int_0^4 \left(x+3x^3\right)dx = \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\frac{4i}{n}+3\left(\frac{4i}{n}\right)^3 = \lim_{n\to\infty}\frac{16}{n^2}\sum_{i=1}^{n} i + \frac{3\cdot 4^4}{n^4}\sum_{i=1}^{n} i^3 = \lim_{n\to\infty}\frac{16\left(n^2+n\right)}{2n^2}+\frac{3\cdot 4^4\left(n^2+n\right)^2}{4n^4}$$

$$= \lim_{n\to\infty} 8+\frac{8}{n}+3\cdot 4^3+\frac{2\cdot 3\cdot 4^3}{n}+\frac{3\cdot 4^3}{n^2} = 8+192=200$$

As a check, $\int_0^4 \left(x+3x^3\right)dx = \left.\frac{x^2}{2}+\frac{3x^4}{4}\right|_0^4 = 8+192=200$

4. Evaluate the integral by interpreting it in terms of familiar area formulae.
   a. $\int_0^1 \left(1+\sqrt{1-x^2}\right)dx$

      SOLN $\int_0^1 \left(1+\sqrt{1-x^2}\right)dx = 1+\frac{\pi}{4}$

   b. $\int_{-2}^1 \left(2-|x|\right)dx$

      SOLN: $\int_{-2}^1 \left(2-|x|\right)dx = 4-\frac{1}{2}=\frac{7}{2}$

5. Use Part 1 or Part 2 of the Fundamental theorem of Calculus to evaluate the expression:
   a. $\frac{d}{dx}\int_0^{\pi}\cos\left(t^2\right)dt$

      SOLN: $\frac{d}{dx}\int_0^{\pi}\cos\left(t^2\right)dt = 0$, since the rate of change of a constant is zero.

   b. $\int_0^{\pi}\frac{d}{dt}\sin\left(t^2\right)dt$

      SOLN: $\int_0^{\pi}\frac{d}{dt}\sin\left(t^2\right)dt = \left.\sin\left(t^2\right)\right|_0^{\pi} = \sin\left(\pi^2\right)$

   c. $\frac{d}{dx}\int_{x^2}^{x^3}\left(t^2\right)dt$

      SOLN: $\frac{d}{dx}\int_{x^2}^{x^3}\left(t^2\right)dt = \frac{d}{dx}\int_{x^2}^{0}\left(t^2\right)dt+\int_0^{x^3}\left(t^2\right)dt = -\frac{du}{dx}\frac{d}{du}\int_0^{u}\left(t^2\right)dt+\frac{dv}{dx}\frac{d}{dv}\int_0^{v}\left(t^2\right)dt$

      $$= -2x\left(x^2\right)^2+3x^2\left(x^3\right)^2 = \boxed{3x^8-2x^5}$$

6. Use substitution to simplify the following definite integrals. For each, explicitly state what the components of your substitution are (on separate paper):
   a. $\int_0^{\pi/4}\tan^3\theta\sec^2\theta\, d\theta$ $\quad u = \tan\theta \quad du = \sec^2\theta d\theta$

      $\int_0^{\pi/4}\tan^3\theta\sec^2\theta\, d\theta = \int_0^1 u^3 du = \left.\frac{u^4}{4}\right|_0^1 = \frac{1}{4}$

   b. $\int_0^1 x^2\left(x^3+1\right)^8 dx$ $\quad u = x^3+1 \quad du = 3x^2 dx$

      $\int_0^1 x^2\left(x^3+1\right)^8 dx = \frac{1}{3}\int_1^2 u^8 du = \left.\frac{u^9}{27}\right|_1^2 = \frac{512}{27}-\frac{1}{27}=\frac{511}{27}$