Math 1B §5.6 Homework Problems

1. Simplify the class of antiderivatives for the indefinite integral $\int x^2 \sin ax\,dx$.
2. Simplify the class of antiderivatives for the indefinite integral $\int z^3 e^z\,dz$.
3. Simplify the class of antiderivatives for the indefinite integral $\int e^{-x}\cos x\,dx$.
4. Simplify the class of antiderivatives for the indefinite integral $\int_1^4 \sqrt{t}\ln t\,dt$.
5. Simplify the class of antiderivatives for the indefinite integral $\int_0^{1/2} \sin^{-1} t\,dt$
6. Simplify the class of antiderivatives for the indefinite integral $\int_{\pi/4}^{\pi/2} x\sec^2 x\,dx$.
7. Simplify the class of antiderivatives for the indefinite integral $\int x\tan^{-1} x\,dx$.
8. First make a substitution, then use integration by parts to simplify $\int x^5 \sin\left(x^3\right)dx$.
9. First make a substitution, then use integration by parts to evaluate $\int_1^4 e^{\sqrt{x}}\,dx$.
10. You may find a graphing device useful here. Graph both the function the function $f(x) = x^3 e^{x^2}$ and the antiderivative $F(x) = \int_0^x t^3 e^{t^2}\,dt$
11. a) Prove the reduction formula $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$

    b) Use part (a) to evaluate $\int \cos^2 x\,dx$

    c) Use parts (a) and (b) to evaluate $\int \cos^4 x\,dx$
12. Use a reduction formula to shot that, for odd powers of sine,
    $\int_0^{\pi/2} \sin^{2n+1} x\,dx = \frac{2\cdot 4\cdot 6\cdot\cdots\cdot 2n}{3\cdot 5\cdot 7\cdot\cdots\cdot(2n+1)}$
13. Prove that for even powers of sine, $\int_0^{\pi/2} \sin^{2n} x\,dx = \frac{1\cdot 3\cdot 5\cdot 7\cdot\cdots\cdot(2n-1)}{2\cdot 4\cdot 6\cdot\cdots\cdot 2n}\frac{\pi}{2}$
14. Let $I_n = \int_0^{\pi/2} \sin^n x\,dx$

    (a) Show that $I_{2n+2} \le I_{2n+1} \le I_{2n}$

    (b) Use the result of exercise 12 to show that $\frac{I_{2n+2}}{I_{2n}} \le \frac{2n+1}{2n+2}$

    (c) Use parts (a) and (b) show that $\frac{2n+1}{2n+2} \le \frac{I_{2n+1}}{I_{2n}} \le 1$ and deduce that $\lim_{n\to 0}\frac{I_{2n+1}}{I_{2n}} = 1$

    (d) Use part (c) and Exercises 11 and 12 to show that

    $$\lim_{n\to 0}\frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdot\cdots\cdot\frac{2n}{2n-1}\cdot\frac{2n}{2n+1} = \frac{\pi}{2}$$

    (e) We construct rectangles as follows. Start with a square of area 1 and attach rectangles of area 1 alternately beside or on top of the previous rectangle (see the figure). Find the limit of the ratios of width to height of these rectangles.