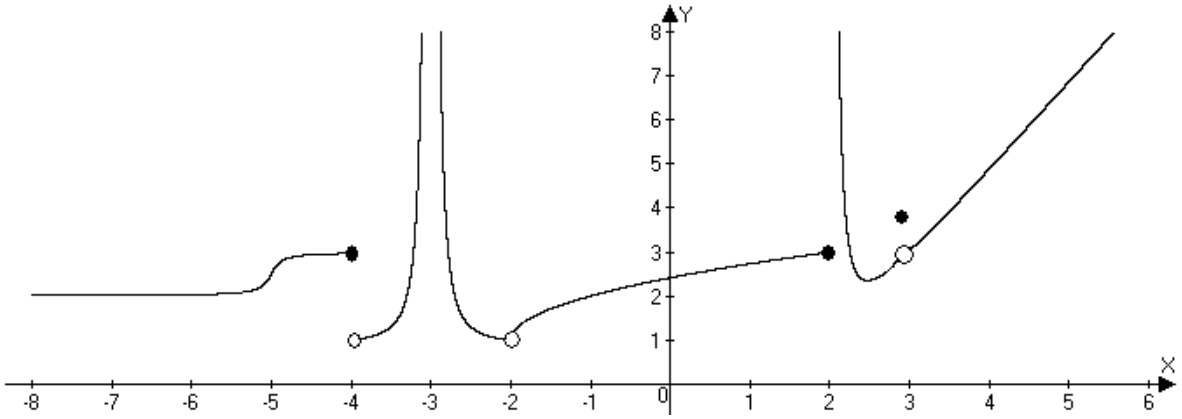
Math 1A – Chapter 2 Test Review Problems.

1. The graph of f is given.



a. Find each limit, or explain why it doesn't exist:

i. $\lim_{x\to -4^-} f(x)$ ii. $\lim_{x\to -4^+} f(x)$ iii. $\lim_{x\to -4} f(x)$

iv. $\lim_{x\to -2} f(x)$ v. $\lim_{x\to 2^+} f(x)$ vi. $\lim_{x\to 3} f(x)$

b. Where does the function have removable discontinuities?

c. Where does the function have a jump discontinuity?

2. Find the limit

a. $\lim_{x\to 3} \dfrac{x^3-27}{x^4-81}$ b. $\lim_{x\to\infty} \dfrac{x-2}{\sqrt{3x^2-x}}$ c. $\lim_{x\to\infty} \dfrac{1}{1-e^{-x}}$ d. $\lim_{x\to 1^-} \dfrac{1}{\ln|x-1|}$

3. Use the intermediate value theorem to prove that $x^2 = 2^x$ has a solution in (-1,0).

4. If the tangent to $y = f(x)$ at (5,4) passes through the point (1,2), find $f(5)$ and $f'(5)$.

5. Find the derivative of $f(x) = \dfrac{3+x}{1-3x}$ using the definition of the derivative.

6. Suppose that we don't have a formula for $g(x)$ but we know that $g(2) = -4$ and $g'(x) = \sqrt{x^2+5}$ for all x.

   a. Use a linear approximation to estimate $g(1.95)$ and $g(2.05)$.

   b. Are your estimates in part (a) too large or too small? Explain.

7. Is there a number a such that $\lim_{x\to -2} \dfrac{2x^2+ax+a}{x^2+x-2}$ exists? If not, why not? If so, find the value of a and the value of the limit.

8. Consider $\lim_{x\to 0} \ln(x+\cos(x))$.

   a. What theorem is essential to evaluating this limit. Why are the conditions of the theorem met?

   b. Use the theorem to evaluate the limit.

9. For the function $f(x)$ whose derivative function $f'(x)$ is graphed below, find where:

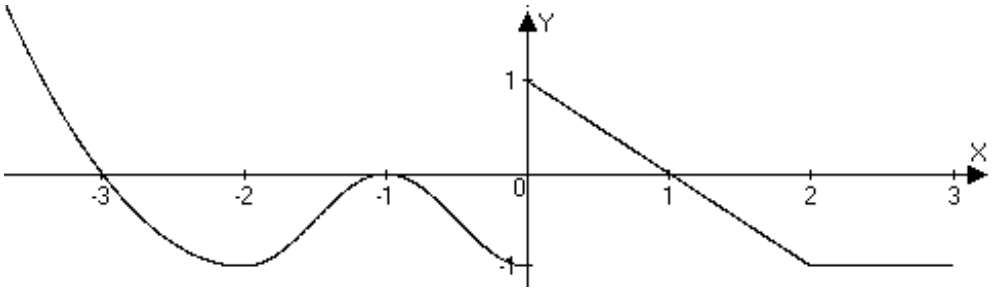
a. $f(x)$ is increasing

b. $f(x)$ is concave up

c. $f(x)$ has a local maximum.

d. $f''(x)$ is positive.

e. $f''(x)=0$.



10. Consider $f(x)=\dfrac{\sqrt{x}-8}{\sqrt[3]{x}-4}$

a. Approximate the value of $f(x)$ at x = 64.001 and 63.999 – what do your results suggest about $\lim_{x\to 64} f(x)$?

b. How close does x have to be to 64 to ensure that the function is within 0.1 of it's limit?

11. Is there a number a such that $\lim_{x\to 1}\dfrac{2x^2+ax+a}{x^2+x-2}$ exists? If not, why not? If so, find the value of a and the value of the limit.

12. Consider $\lim_{x\to \pi/2}\cos\left(2x+\cos(3x)\right)$.

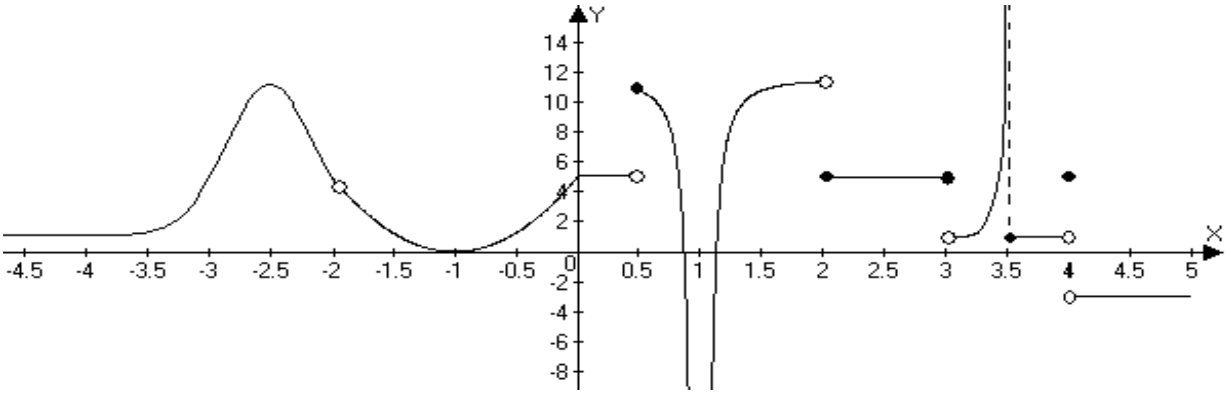
a. What theorem is essential to evaluating this limit. Why are the conditions of the theorem met?

b. Use the theorem to evaluate the limit.

13. For the function g whose graph is shown, approximate the following, writing "DNE" if the limit doesn't exist and ∞ or $-\infty$, as appropriate.

a. $\lim_{x\to -\infty} g(x)$ b. $\lim_{x\to -2} g(x)$ c. $\lim_{x\to 0.5^+} g(x)$

d. $\lim_{x\to 1} g(x)$ e. $\lim_{x\to 2^-} g(x)$ f. $\lim_{x\to 3.5^+} g(x)$



14. Suppose the height H of an object (in meters) at time t (in seconds) is given by

$$H(t)=\begin{cases}0 & \text{if} \quad t<0\\ 1 & \text{if} \quad t\geq 0\end{cases}$$

a. What is the average velocity over the interval $-1\leq t\leq 1$

b. Find an interval over which the average velocity of the object is a 1000 m/s.

15. Let $B(t)$ be the number of Elbonian buffalo per capita at time t. The table below gives values of $B(t)$ as of June 30 of the specified year. What is your best approximation to the value of $B'(2000)$?

t	1998	1999	2000	2001	2002
$B(t)$	12.5	10.2	9.80	9.20	7.95

16. Consider the function $x(t)=\dfrac{1}{1+t^2}$.

a. Use the definition of the derivative to show that $x'(t)=\dfrac{-2t}{\left(1+t^2\right)^2}$.

b. Find an equation for the line tangent to $x(t)$ where $t = 1$.

c. Use a linear approximation to approximate $x(1.05)$

17. For the function $f(x)$ whose derivative function $f'(x)$ is graphed below, find where:

a. $f(x)$ is increasing

b. $f(x)$ is concave up

c. $f(x)$ has a local maximum.

d. $f''(x)$ is positive.

e. $f''(x)=0$.