Math 1A **Name (Print):** _____

Chapter 3 Test

Write all responses on separate paper. Show your work in detail for credit. No calculators.

1. (18 points) Calculate $\dfrac{dy}{dx}$

   (a) $y = \dfrac{1}{\sqrt[3]{x}} - \dfrac{1}{\sqrt[4]{x^3}}$

   (b) $y = \dfrac{\sin(2x)}{1-\tan(2x)}$

   (c) $y = e^{-x^2}\sin(3x)$

   (d) $\arcsin(\sqrt{x})$

   (e) $y = \cosh(x^2)10^{-x^2}$

   (f) $\ln\left|\dfrac{1+x^2}{1-x^2}\right|$

2. (10 points) Use the **definition** of the derivative (that is, $f'(x) \equiv \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$) to compute $\dfrac{d}{dx}\sec(x)$

   Hint: $\lim_{h\to 0}\dfrac{\sin h}{h}$ (numerator $\to 1$) and $\lim_{h\to 0}\dfrac{\cos(h)-1}{h}$ (numerator $\to 0$)

3. (10 points) Find an equation for the line tangent to the curve given by $xy^2 + yx^2 = 6$ at the point $(1, -3)$

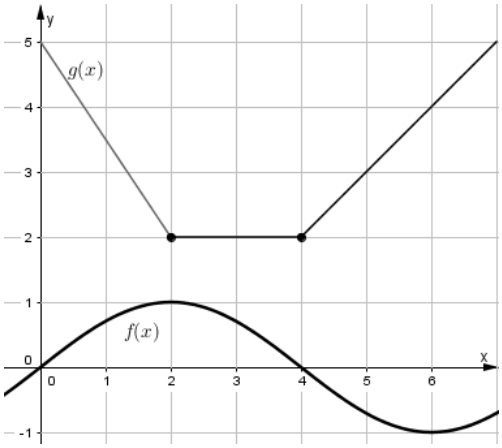
4. (10 points) Find two points on the curve $y = 3\arctan(2x)$ where the line tangent to the curve is parallel to $17y - 6x = 0$.

5. (12 points) If f and g are the functions whose graphs are shown, where $f(x)$ is sinusoidal function, $f(x) = \sin\left(\dfrac{\pi x}{4}\right)$.
   Let $P(x) = f(x)g(x), Q(x) = f(x)/g(x)$ and $C(x) = f(g(x))$. Find

   (a) $P'(1)$

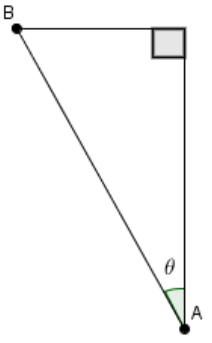
   (b) $Q'(3)$

   (c) $C'(6)$



6. (8 points) The volume of a sphere is increasing at a rate of 4 cubic meters per hour. How fast is the surface area increasing when the radius 50 cm?

7. (10 points) Two students A and B are walking on straight roads that meets at right angles. Student A moves away from the intersection at 1 m/sec and student B approaches that intersection at 2 m/sec as shown in the figure. At what rate is the angle θ changing when A is 20 m from the intersection and B is 10 m from the intersection? Express your answer in radians per second.
   Hint: To work this problem, it helps to introduce variables to represent the lengths of the sides and then express $\tan(\theta)$ in terms of those variables.



8. (10 points) A population of plankton experiences natural growth. On day zero the population has a mass of 4 grams. On day 10 the population has a mass of 100 grams. Find an expression for the population mass on day 15.

9. (12 points) Find the linearization of the function $f(x) = \sqrt[3]{1+x}$ at $a = 0$ and use it to approximate the numbers $\sqrt[3]{0.95}$ and $\sqrt[3]{1.1}$