

Write all responses on separate paper. Show your work in detail for credit. No calculators.

1. (18 points) Compute

(a) $\lim_{x \rightarrow 0} \frac{(1+x)^{1/3} - (1 + \frac{1}{3}x)}{x^2}$

(b) $\lim_{n \rightarrow \infty} \frac{n^3 + 3n^2}{2n^3 - n}$

(c) $\lim_{x \rightarrow 0^+} \frac{1}{x} - \frac{1}{\sin x}$

2. (15 points) Let $f(x) = 2x^3 - 12x^2 + 18x - 2$ be a cubic polynomial with zeros $x_1 \approx 0.12, x_2 \approx 2.35, x_3 \approx 3.53$

(a) Find the critical points of the function.

(b) Find the inflection point.

(c) Construct a careful graph of the function showing these features.

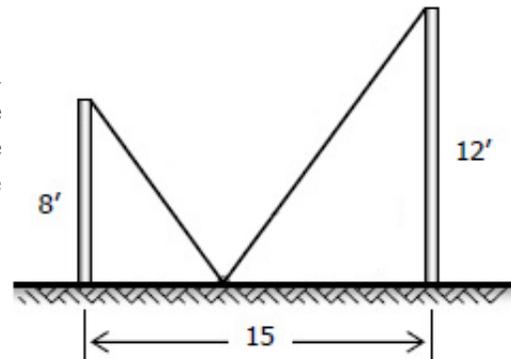
3. (15 points) Let $f(\theta) = 2 \sin \theta + \sin 2\theta$ on $\theta \in [0, 2\pi]$

(a) Find the intercepts of the function.

(b) Find the extreme values of the function.

(c) Find the inflection points.

4. (19 points) The figure at right shows two vertical posts, 8 and 12 feet high, which stand 15 feet apart. A wire is to join the tops of these posts and a stake on the ground between them. Use calculus to determine where to place the stake to minimize the length of wire needed?



Start by introducing a control variable.

5. (18 points) Find a number $c \in (A, B)$ such that $f'(c) = \frac{f(B) - f(A)}{B - A}$ where

(a) $f(x) = x^{2/3}$ and $A = 0, B = 1$

(b) $f(x) = x + \frac{1}{x}$ and $A = \frac{1}{3}, B = 3$

6. (15 points) We will use Newton's method to find $\sqrt[3]{A}$

(a) Show $x = \sqrt[3]{A}$ is a root of $f(x) = x^3 - A$

(b) Show Newton's method applied to $f(x)$ leads to the iterative formula $x_{n+1} = \frac{2x_n + A/x_n^2}{3}$

(c) Show that if $A = 8$ and $x_1 = 1$ then $x_3 = \frac{554}{225}$