Math 15 – Discrete Structures – Homework 18 Solutions – 5.1

5.1#18: The following is known as the Ackerman function.

```
function Ack(m, n ∈ {0,1,2, ...})
   if m = 0 then
      return n + 1
   else
      if n = 0 then
         return Ack(m − 1,1)
      else
         return Ack(m − 1, Ack(m, n − 1))
```

Compute the values returned by the following function calls.
(a) Ack(0,7)= 8
(b) Ack(1,0) = Ack(0,1) = 2
(c) Ack(1,1) = Ack(0,Ack(1,0)) = 1+Ack(1,0) = 3
(d) Ack(2,1) = Ack(1,Ack(2,0)) = Ack(1,Ack(1,1)) = Ack(1,Ack(0,Ack(1,0))) = Ack(1,Ack(0,2)) = Ack(1,3) = Ack(0,Ack(1,2)) = Ack(0,Ack(0,Ack(1,1))) = Ack(0,Ack(0,3)) = Ack(0,4) = 5.

5.1#22: Write a recursive function in pseudocode that computes the value of the following recurrence relation: $H(n) = \begin{cases} 1, & \text{if } n = 1 \\ H(n-1) + 6n - 6, & \text{if } n > 1 \end{cases}$. Give descriptive preconditions and postconditions.

SOLN:

Predcondition: $n \in \{1,2,3, \dots\}$
Postcondition: $3n^2 - 3n + 1$

```
function H(n ∈ {1,2,3, ...})
  if n = 1 then
    return 1
  else
    return 6(n − 1) + H(n − 1)
```