Math 5 – Trigonometry – Chapter 4 Test Review– fall ’06

Outline:

- 4.1: The Unit Circle $x^2 + y^2 = 1$.
    - Given the one of the coordinates of a point on the circle and the quadrant of the point, find the coordinates of the other point.
    - Find the terminal point $P(x,y)$ corresponding to an arclength t on the circle extending either counterclockwise (positive direction) or clockwise (negative direction) from (1,0) including the standard positions where t is either a multiple of $\frac{\pi}{6}$ or $\frac{\pi}{4}$.
    - Find the reference number $\overline{t}$ corresponding to any unit circle arclength t, including $|t| > 2\pi$.
    - Use the reference number to find terminal points.

- 4.2: The Trigonometric Functions of Real Numbers
    - Definitions of the six trig functions of arclength t from (1,0) on the unit circle in terms of the coordinates of the terminal point $P(x,y)$.
    - Relationship to the trigonometric functions of angles and the radian measure of an angle (page 239)
    - Domain and range of the trigonometric functions.
    - Signs of the trigonometric functions as determined by the quadrant of the terminal point $P(x,y)$.
    - Reciprocal identities.
    - Even and odd properties of trigonometric functions.
    - Pythagorean Identities.
    - Using the identities to write on trig function in terms of another.

- 4.3: Trigonometric Graphs
    - Periodic properties of sine and cosine.
    - Transformations of sine and cosine.
    - Amplitude and period from vertical stretch/shrink and horizontal stretch/shrink.
    - Phase shift from horizontal shift.
    - Graphing sums of sine and cosine with different periods.
    - Decaying and variable amplitudes.
    - Oscillation inside an envelope: $y = A(t)\sin(b(t-c))$

- 4.4: More Trigonometric Graphs
    - Periodic properties of tan, sec, cot and csc functions.
    - Period of $f(x) = \tan(b(x-c))$ and $g(x) = \cot(b(x-c))$
    - Period of $f(x) = \sec(b(x-c))$ and $g(x) = \csc(b(x-c))$
    - Graphing $f(x) = A\sec(b(x-c))$ and $g(x) = A\csc(b(x-c))$

- 4.5: Modeling Harmonic Motion
    - Simple Harmonic Motion
    - Damped Harmonic Motion

1. Express the arclength, t, on the unit circle of an angle swept out by rotating the positive x axis 36° about the circle's center in the counterclockwise direction. Use a calculator to approximate the coordinates of the terminal point $P(x,y)$ corresponding to this t to the nearest ten thousandth.

2. Consider the point $P(x,y)$ on the unit circle corresponding to an angle with radian measure $t = \frac{5\pi}{6}$.
   a. What is the degree measure of this angle?
   b. What is the degree measure of a supplementary angle (supplementary angles sum to 180°).
   c. What is the degree measure of a complementary angle (supplementary angles sum to 90°).
   d. Find exact values for each of the following and illustrate its position on the unit circle:

      i. $\cos(t)$ ii. $\cos(t+\pi)$ iii $\cos(t-\pi)$ iv. $\cos\left(t-\frac{\pi}{2}\right)$ v. $\cos\left(t+\frac{\pi}{2}\right)$

3. Suppose a terminal point determined by t is the point $P(x,y) = \left(-\frac{7}{25}, -\frac{24}{25}\right)$.
   a. Verify that the point lies on the unit circle.
   b. What are the coordinates of the terminal point for $t+\pi$?
   c. What are the coordinates of the terminal point for $t+\frac{\pi}{2}$?

4. Suppose a terminal point determined by t is $P(x,y)$ on the unit circle, where $\frac{y}{x} = -\frac{15}{8}$.
   a. What quadrants could P be in?
   b. What are the absolute values of the coordinates of x and y?
   c. Find exact representations for the values of $\csc(t)$ and $\cot(t)$.

5. Suppose a terminal point $P(x,y)$ in QIV on the unit circle has y-coordinate $-\frac{\sqrt{11}}{5}$. Find
   a. $\sec(t)$ b. $\tan(t)$

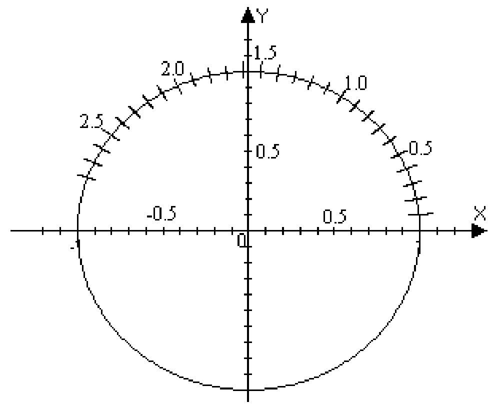
6. Find the reference number for each and plot its position on the unit circle together with exact values (in simplest radical form) for the x and y coordinates of the point.
   a. $t = \frac{53\pi}{6}$ b. $t = \frac{53\pi}{4}$

7. Find the period and equations for at least two asymptotes and graph the function $f(t) = 1 + \tan(3t+1)$. Sketch a graph showing the functions intercepts and how the function approaches the asymptotes.

8. Suppose a terminal point $P(x,y)$ on the unit circle has $y = \frac{12}{37}$. What are two different possible values for x? What quadrants are these in?

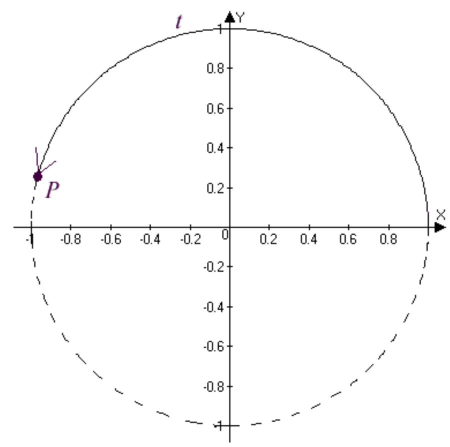
   a.

9. Suppose that $1 \le t \le 2.5$. Estimate the corresponding intervals for the values of cos(t) and sin(t) and highlight these on the diagram at right:



10. Find the amplitude, period and phase shift of the $W(t) = 2 + 2\sin\left(2\pi t - \frac{\pi}{6}\right)$ and sketch a graph showing at least one wave form. Be careful scale and label axes in your graph.

11. Suppose a terminal point $P(x,y)$ as shown at a distance t along the unit circle has y-coordinate $\frac{\sqrt{5}}{9}$. Find
    a. $\sin(t)$
    b. $\tan(t)$



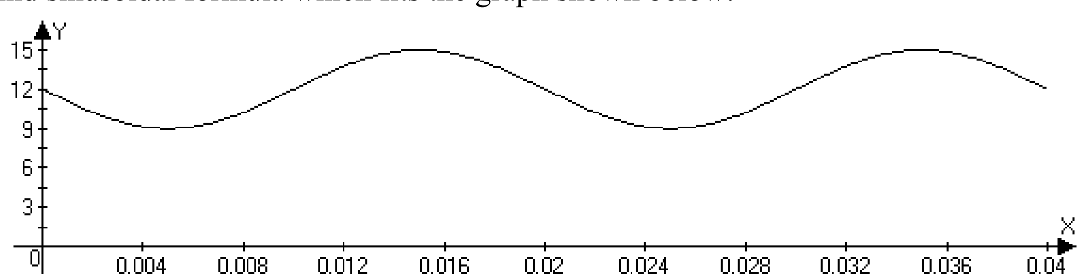
12. Find an exact value for each of the following and show its position on the unit circle.
    a. $\sin\left(\frac{5\pi}{4}\right)$
    b. $\tan\left(\frac{5\pi}{6}\right)$

13. Express $\cos(t)$ in terms of $\csc(t)$, if the terminal point is in quadrant IV.

14. Find the amplitude, period and phase shift of the $W(t) = 117\sin\left(120\pi t - \frac{\pi}{2}\right)$ and sketch a graph showing at least one wave form. Be careful scale and label axes in your graph.

15. Find the period and at least two asymptotes and graph the function $f(t) = \frac{\tan(4t)}{\sqrt{3}}$.

16. Find sinusoidal formula which fits the graph shown below:



17. Consider the function $f(x) = 2e^{-x}\cos(4x)$. Sketch graphs for $y = 2e^{-x}$, $y = -2e^{-x}$ and $y = f(x)$ together showing two oscillations of the cosine function between these curves.