Math 5 – Trigonometry – Chapter 6 Test – fall '12 Name_________________________
Show your work for credit. Write all responses on separate paper.

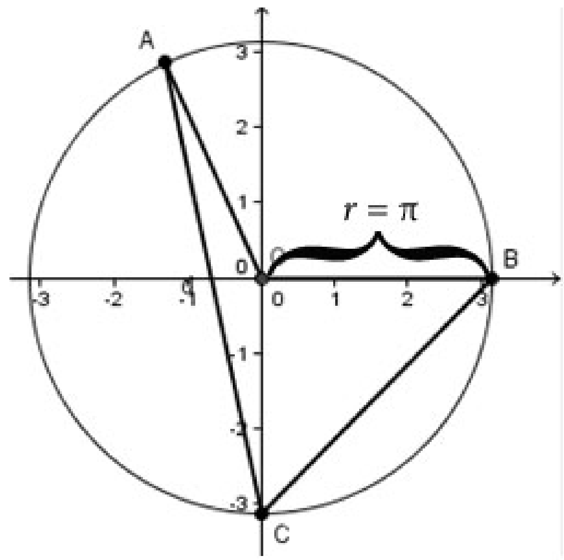
1. Consider an arclength of 2π along the circumference of a circle of radius π as shown in the diagram at right.

   a. What is the radian measure of $\angle BOA$?

   b. What is the radian measure of $\angle BCA$?

   c. What is the area of sector $\overset{\frown}{BOA}$?

   d. What is the area of quadrilateral $BOAC$?



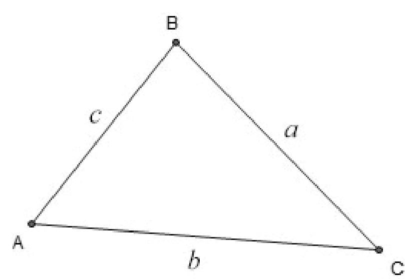
2. What is the radian measure of the central angle of a sector of area 17π in a circle of radius $4\sqrt{2}$?

3. Assume the earth revolves around the sun once every 365 days and that the orbit is circular with the distance from the earth to the sun at 150 million kilometers.
   a. Find the angular velocity of a person standing on the north pole in radians per second (ignore the rotation of earth on axis).
   b. Find the linear velocity in meters/second of a person on the north pole relative to the sun. Use 1000 meter = 1 km. Show all the factor labeling in your computation carefully.

4. Suppose that in acute ΔABC shown at right, $\angle C \approx 41.40°$, $a = 5$ and $c = 4$.

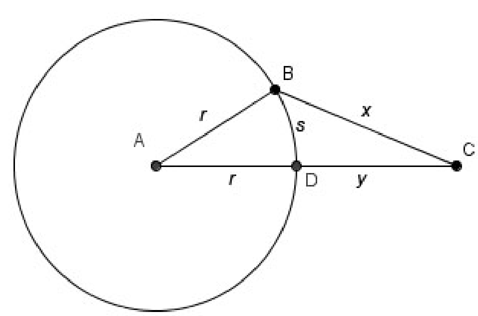
   a. What is the value of b to the nearest hundredth?

   b. There is an obtuse triangle with the same values for a, c and $\angle C$, solve it.



5. In triangle ABC, $\angle A = 45°$, $\angle C = 120°$ and $a = 24$, find the exact value for c in simplest radical form.

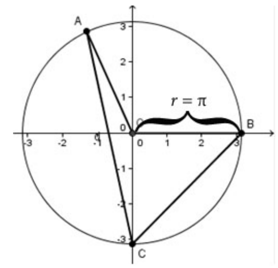
6. The circle in the figure at right has radius r and center A. The distance from B to C is x, the distance from C to D is y and the length of the $arc\ BD$ is s, as labeled.
   Find s if $r = 5$, $y = 6$ and $x = 8$.



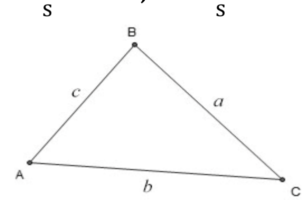
7. The diagonals of a parallelogram are 56 cm and 34 cm and intersect at an angle of 120º. Find the length of the shorter side.

Math 5 – Trigonometry – Chapter 6 Test Solutions – fall '12

1. Consider an arclength of 2π along the circumference of a circle of radius π as shown in the diagram at right.

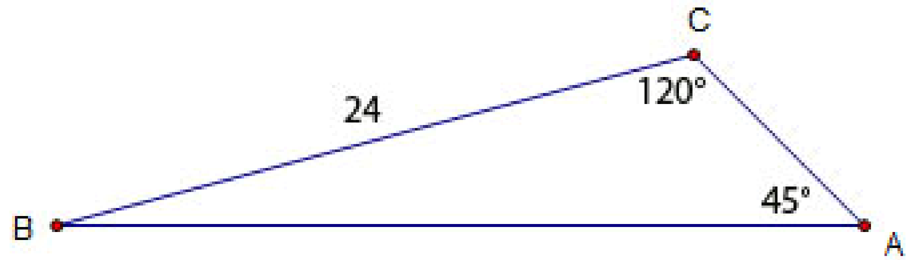


   a. What is the radian measure of $\angle BOA$?
   SOLN: Plug into the formula $s = r\theta$ to get $2\pi = \pi\theta$ to see that $\theta = 2$ radians.
   b. What is the radian measure of $\angle BCA$?
   SOLN: Since $\angle BCA$ subtends a central angle of 2 rads, it subtends a circular angle of 1 radian, by Euclid, Book III, proposition 20.
   c. What is the area of sector $\overset{\frown}{BOA}$?
   SOLN: The formula $A = \frac{r^2\theta}{2}$, derived by supposing the area is jointly proportional to the square of the radius and the central angle, leads to $A = \frac{\pi^2 2}{2} = \pi^2$ square units.
   d. What is the area of quadrilateral $BOAC$?
   SOLN: There are a variety of ways to do this. The general idea is to divide and conquer, say, by dividing the area of $BOAC$ into triangles ΔBOC and ΔAOC. ΔBOC is an isosceles right triangle, so the area is simply $\pi^2/2$.
   ΔAOC is also isosceles, with central angle $\frac{3\pi}{2} - 2$ so its area is
   $A = \frac{1}{2}absin\theta = \frac{1}{2}\pi^2 \sin\left(\frac{3\pi}{2} - 2\right) \approx 2.0536$ square units
2. What is the radian measure of the central angle of a sector of area 17π in a circle of radius $4\sqrt{2}$?
   SOLN: $A = 17\pi = \frac{r^2\theta}{2} = 16\theta \Rightarrow \theta = \frac{17\pi}{16}$
3. Assume the earth revolves around the sun once every 365 days and that the orbit is circular with the distance from the earth to the sun at 150 million kilometers.
   a. Find the angular velocity of a person standing on the north pole in radians per second (ignore the rotation of earth on axis). SOLN:
   $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{365}\frac{\text{rad}}{\text{day}} \times \frac{1\text{ day}}{24\text{ hrs}} = \frac{\pi}{4380}\frac{\text{rad}}{\text{hr}} \times \frac{1\text{ hr}}{60\text{ min}} = \frac{\pi}{262800}\frac{\text{rad}}{\text{min}} \times \frac{1\text{ min}}{60\text{ sec}} = \frac{\pi}{15768000}\frac{\text{rad}}{\text{sec}}$
   b. Find the linear velocity in meters/second of a person on the north pole relative to the sun. Use 1000 meter = 1 km. Show all the factor labeling in your computation carefully. SOLN:
   $v = \omega r = \frac{\pi}{15768000}\frac{\text{rad}}{\text{sec}} \times 150 \times 10^6\text{km} \times \frac{1000\text{m}}{\text{km}} = \frac{1.5\times 10^{11}\pi}{1.5768\times 10^7}\frac{\text{m}}{\text{s}} \approx 9519\pi\frac{\text{m}}{\text{s}} \approx 29{,}900\frac{\text{m}}{\text{s}}$
4. Suppose that in acute ΔABC shown at right, $\angle C \approx 41.40°$, $a = 5$ and $c = 4$.



   a. What is the value of b to the nearest hundredth?
   SOLN: This is an ASS situation, which since $5 \cdot \sin(41.4°) \approx 3.3 < c < a$ has a unique acute triangle solution. By the law of signs,
   $\angle A = \sin^{-1}\left(\frac{a \cdot \sin C}{c}\right) = \sin^{-1}\frac{3.3066}{4} \approx 55.7551°$ Thus $\angle B \approx 180° - 41{,}4° - 55.7551° = 82.8449°$. Then the law of signs yields $b = c \cdot \frac{\sin B}{\sin C} \approx 4 \cdot \frac{\sin 82.8449°}{\sin 41.4°} \approx 6.001$
   b. There is an obtuse triangle with the same values for a, c and $\angle C$, solve it.
   SOLN: Instead of taking the acute angle at A, we can instead take the supplement, $180° - \angle A = 124.2449°$ (note that this is another solution to $\sin A = \frac{a \cdot \sin C}{c}$)
   Then $\angle B \approx 180° - 41{,}4° - 124.2449° = 14.3551°$ and $b \approx 4 \cdot \frac{\sin 14.3551°}{\sin 41.4°} \approx 1.500$

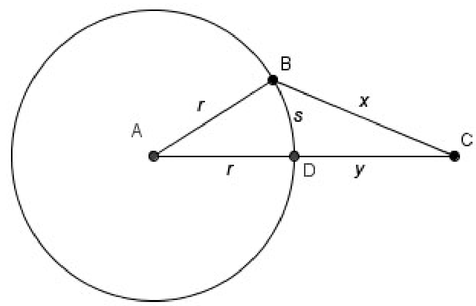
5. In triangle ABC, $\angle A = 45°$, $\angle C = 120°$ and $a = 24$, find the exact value for c in simplest radical form.
SOLN: The given information is shown in the diagram below. By the law of signs, $\frac{c}{\sin 120°} = \frac{24}{\sin 45°}$ so that $c = \frac{\sqrt{3}}{2} \cdot \sqrt{2} \cdot 24 = 12\sqrt{6}$.



6. The circle in the figure at right has radius r and center A. The distance from B to C is x, the distance from C to D is y and the length of the $arc\ BD$ is s, as labeled.
Find s if $r = 5$, $y = 6$ and $x = 8$.
SOLN: The strategy here is to first use the law of cosines to find $\angle A$ and then use the definition of radian measure to find s.
By the law of cosines,

$$\angle A = \cos^{-1}\left(\frac{5^2 + 11^2 - 8^2}{2 \cdot 5 \cdot 11}\right) = \cos^{-1}\frac{41}{55} \approx 41.80°$$

In radians, $\angle A \approx 0.7296 = \frac{s}{r} \Leftrightarrow s \approx 5 \cdot 0.7296 = 3.6479$



7. The diagonals of a parallelogram are 56 cm and 34 cm and intersect at an angle of 120º. Find the length of the shorter side.
SOLN: This is a SAS situation, as illustrated. So the length of the shorter side can be found using the law of cosines: $\sqrt{17^2 + 28^2 - 2 \cdot 17 \cdot 28 \cos 60°} = \sqrt{289 + 784 - 476} = \sqrt{597}$