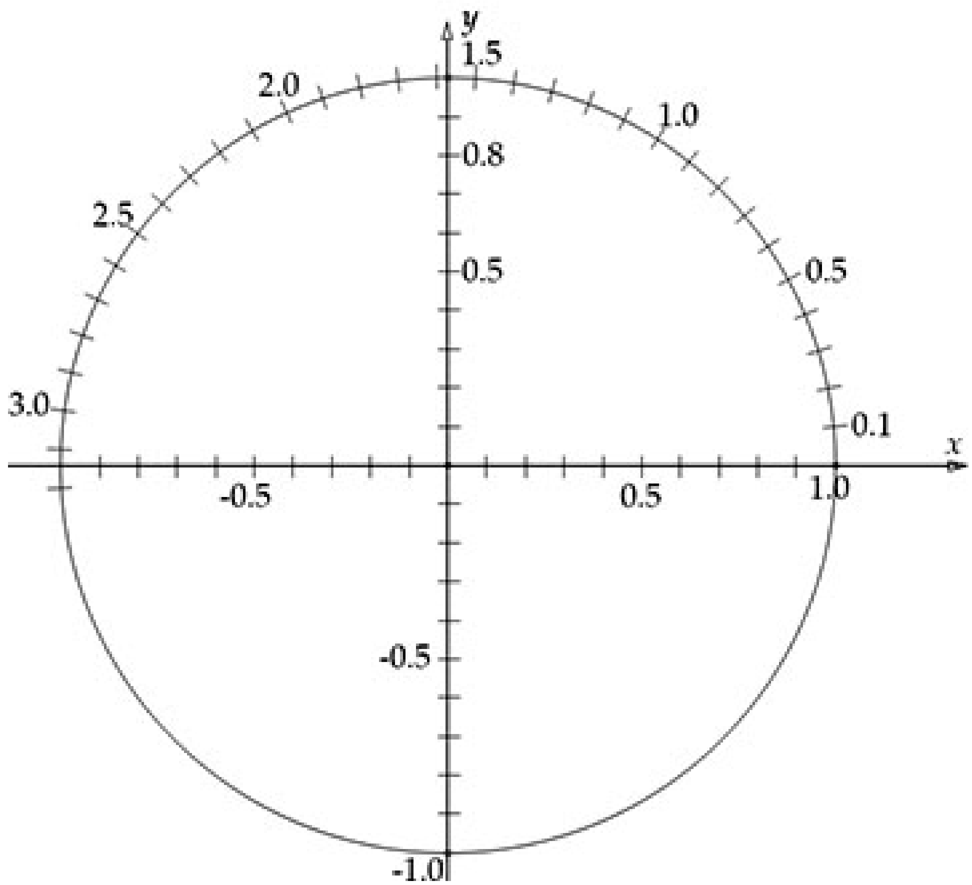
Math 5 – Trigonometry – Chapter 5 Test – fall '12 Name__________________________
Show your work for credit. Write all responses on separate paper. Do not use an electronic calculator.

1. If the arclength $t = \frac{25\pi}{4}$ is traced counterclockwise along the unit circle from (1,0) then
   a. What is the reference number for *t* ?
   b. What are the coordinates of the terminal point *P*(*x*,*y*) ?
   c. Draw the unit circle and plot the terminal point *P*(*x*,*y*).

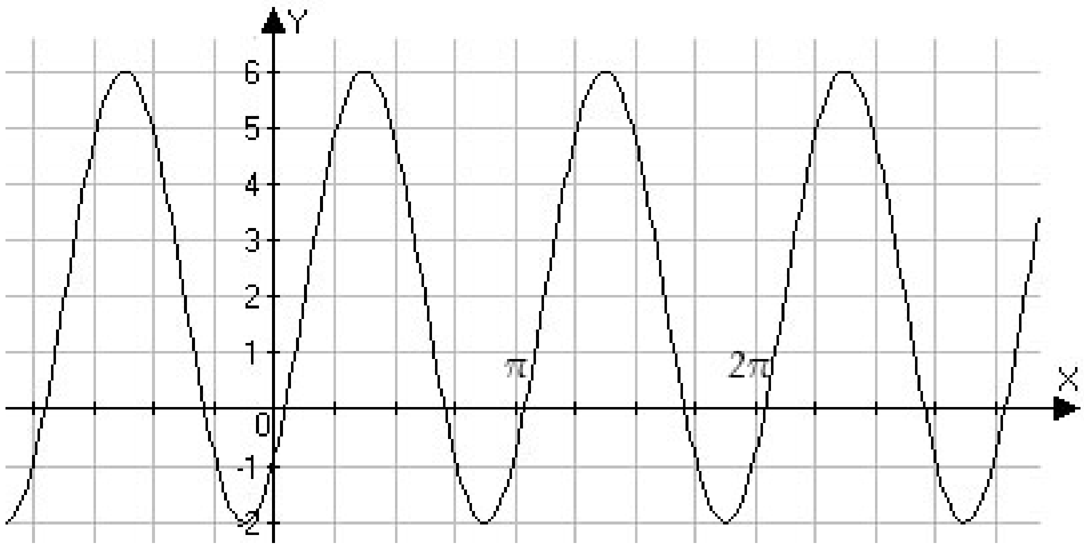
2. Use the unite circle shown at right to answer the following.
   a. Locate the point $\left(-\frac{3}{5}, \frac{4}{5}\right)$ on the circle.
   b. Approximate the smallest positive value of *t* that will have the terminal point $\left(-\frac{3}{5}, \frac{4}{5}\right)$.
   c. Locate the point on the circle in the first quadrant where $x = \frac{24}{25}$. What is the exact value of the *y* coordinate at that point?
   d. Use the diagram at right to approximate to the nearest tenth two value of *t* so that $\sin(t) = \frac{7}{25}$.
   e. Approximate to the nearest tenth the interval for *t* in the first quadrant where $\frac{7}{25} < \cos(t) < \frac{24}{25}$.



3. Suppose that $\sin(t) = \frac{\sqrt{15}}{4}$ and $\cos(t) < 0$.
   Find $\cos(t)$, $\tan(t)$, $\sec(t)$, $\csc(t)$ and $\cot(t)$. Don't bother to approximate.

4. Consider the function, $y = 12 + 4\sin\left(\frac{\pi}{180}(\mathrm{t} - 80)\right)$.
   a. Find the amplitude, period, phase shift and an equation for the line of equilibrium for this sinusoid.
   b. Construct a table of values and a graph for one period of the function,
   clearly showing the positions of at least 5 key points.

5. Find an equation for the sinusoid whose graph is shown:

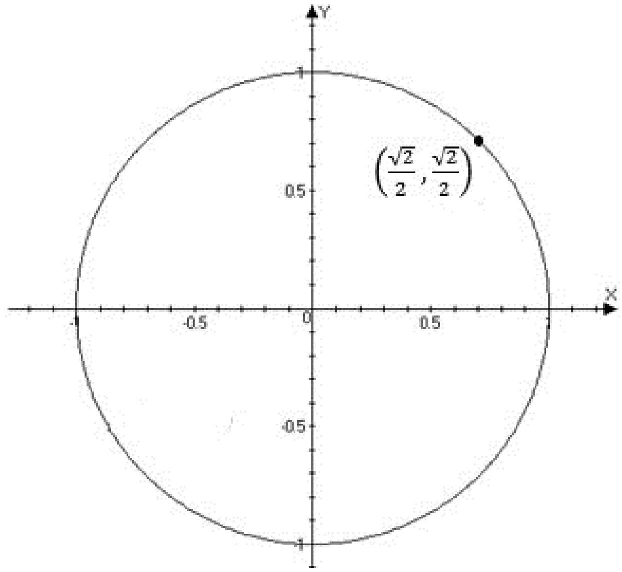


6. Consider the function $f(x) = \sec(2x - 1)$.
   a. Find the equations for three adjacent vertical asymptotes and sketch them in with dashed lines.
   b. Find the *x*-coordinates where $y = \pm 1$.
   c. Carefully construct a graph of the function showing how it passes through the points where $y = -2, y = 2,\ y = \pm 1$ and how it approaches the vertical asymptotes.

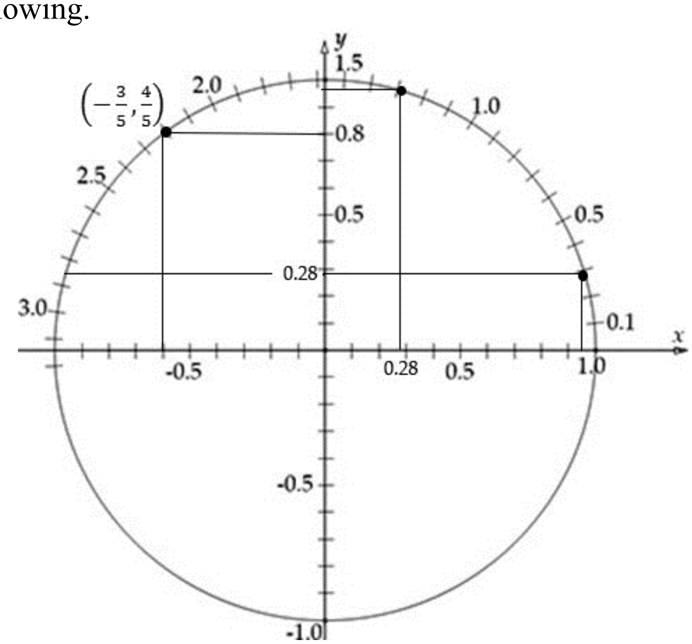
7. Suppose sin *t* = 2/3 and *t* is in the first quadrant. Find the following:
   a. $\cos(t + \pi)$
   b. $\cos\left(t + \frac{\pi}{2}\right)$
   c. $\cos\left(\frac{\pi}{2} - t\right)$

Math 5 – Trigonometry – Chapter 5 Test Solutions – fall '12

1. If the arclength $t = \frac{25\pi}{4}$ is traced counterclockwise along the unit circle from (1,0) then
   a. What is the reference number for *t* ?
   ANS: $\hat{t} = \frac{\pi}{4}$
   b. What are the coordinates of the terminal point *P*(*x*,*y*) ?
   ANS: Since $t = \frac{25\pi}{4} = \frac{24\pi+\pi}{4} = 6\pi + \frac{\pi}{4}$ is three full rotations plus $\frac{1}{8}$ turn to terminate at $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
   c. Draw the unit circle and plot the terminal point *P*(*x*,*y*).Find the coordinates of the terminal point *P*(*x*,*y*).
   ANS: Shown at right.



2. Use the unite circle shown at right to answer the following.
   a. Locate the point $\left(-\frac{3}{5}, \frac{4}{5}\right)$ on the circle. ANS: See diagram →
   b. Approximate the smallest positive value of *t* that will have the terminal point $\left(-\frac{3}{5}, \frac{4}{5}\right)$.
   ANS: This is evident in the diagram ≈ 2.2
   c. Locate the point on the circle in the first quadrant where $x = \frac{24}{25}$. What is the exact value of the *y* coordinate at that point?
   ANS: The equation for the circle is $x^2 + y^2 = 1$. Plug in $x = \frac{24}{25}$ & solve for *y*: $\left(\frac{24}{25}\right)^2 + y^2 = 1$
   $$\Leftrightarrow y^2 = 1 - \frac{576}{625} = \frac{49}{625} \Leftarrow y = \frac{7}{25}$$
   d. Use the diagram at right to approximate to the nearest tenth two value of *t* so that $\sin(t) = \frac{7}{25}$.
   ANS: $\frac{7}{25} = 0.28$ where *t* could be about 0.3 or about $\pi - 0.3 \approx 2.8$ or 2.9
   e. Approximate to the nearest tenth the interval for *t* in the first quadrant where $\frac{7}{25} < \cos(t) < \frac{24}{25}$.
   ANS: Using the symmetry of $\cos(t)$ and $\sin(t)$ about the line *y* = *x* we see that $\frac{7}{25} < \cos(t) < \frac{24}{25} \Leftrightarrow \cos^{-1}\frac{24}{25} < t < \cos^{-1}\frac{7}{25}$. This means that, approximately, $0.3 < t < 1.3$



3. Suppose that $\sin(t) = \frac{\sqrt{15}}{4}$ and $\cos(t) < 0$.

Find $\cos(t)$, $\tan(t)$, $\sec(t)$, $\csc(t)$ and $\cot(t)$. Don't bother to approximate.

ANS: $\cos(t) = -\sqrt{1-\frac{15}{16}} = -\sqrt{\frac{1}{16}} = -\frac{1}{4}$, $\tan(t) = \frac{\sin(t)}{\cos(t)} = -\sqrt{15}$,

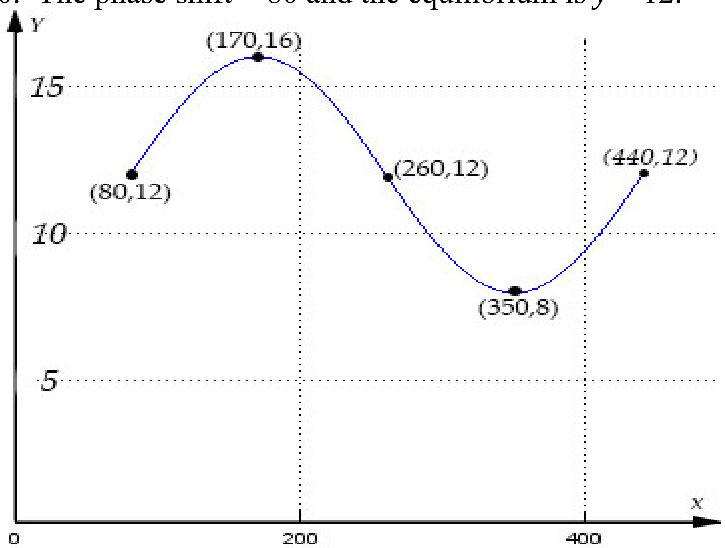
$\sec(t) = \frac{1}{\cos(t)} = -4$, $\cot(t) = \frac{1}{\tan(t)} = -\frac{\sqrt{15}}{15}$ and $\csc(t) = \frac{1}{\sin(t)} = \frac{4\sqrt{15}}{15}$

4. Consider the function, $y = 12 + 4\sin\left(\frac{\pi}{180}(t-80)\right)$.

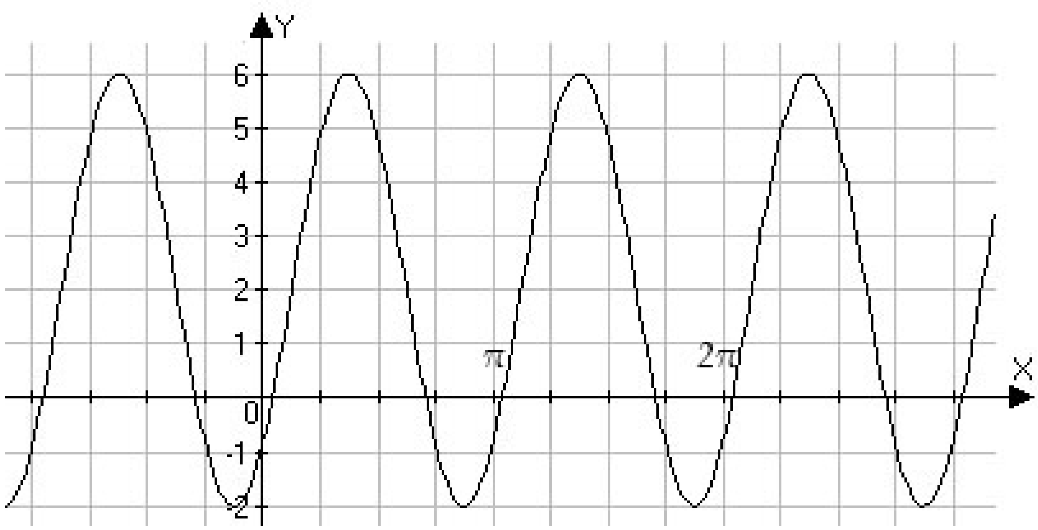
a. Find the amplitude, period, phase shift and an equation for the line of equilibrium for this sinusoid.
ANS: The amplitude is 4. The period = 360. The phase shift = 80 and the equilibrium is $y = 12$.

b. Construct a table of values and a graph for one period of the function, clearly showing the positions of at least 5 key points.

x	80	170	260	350	440
y	12	16	12	8	12



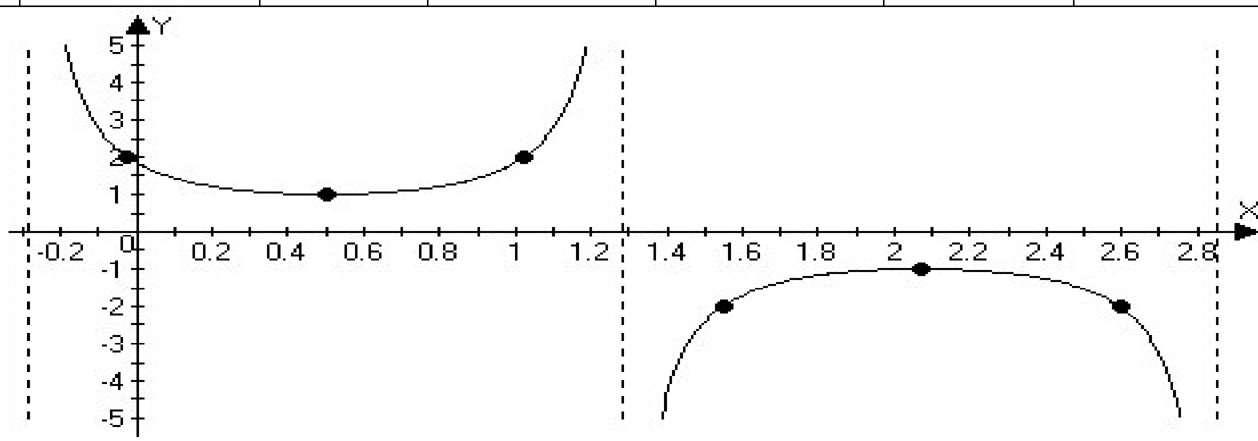
5. Find an equation for the sinusoid whose graph is shown:



ANS: The line of equilibrium is $\frac{-2+6}{2} = 2$ and the amplitude is 4. The period of oscillation is π and the phase shift is $\frac{\pi}{8}$, so the curve is described by the function $y = 2 + 4\sin\left(2x - \frac{\pi}{4}\right)$.

6. Consider the function $f(x) = \sec(2x - 1)$.
   a. Find the equations for three adjacent vertical asymptotes and sketch them in with dashed lines.
   SOLN: $f(x)$ is undefined when $\cos(2x-1) = 0$, leading to division by zero. This is true when $2x - 1 = \pm\frac{\pi}{2} \Leftrightarrow x = \frac{1}{2} \pm \frac{\pi}{4} \approx 0.5 \pm 0.785 = -0.285$ or 1.285. Since the period of the function is $\frac{2\pi}{2} = \pi$, we can add π to $\frac{1}{2} - \frac{\pi}{4}$ to get a third vertical asymptote at $\frac{1}{2} + \frac{3\pi}{4} \approx 2.85$. So, all together, the three adjacent vertical asymptotes are $x = \frac{1}{2} - \frac{\pi}{4} \approx -0.3$, $x = \frac{1}{2} + \frac{\pi}{4} \approx 1.3$ and $x = \frac{1}{2} + \frac{3\pi}{4} \approx 2.9$
   b. Find the *x*-coordinates where $y = \pm 1$.
   ANS: $\sec(2x-1) = 1 \Leftrightarrow \cos(2x-1) = 1$ which means that $2x - 1 = k\pi$, some integer multiple of π. So, $x = \frac{1}{2} + \frac{k\pi}{2}$. For $k = 0$ and $k = 1$ we have $x = \frac{1}{2}$ or $x = \frac{1+\pi}{2} \approx 2.1$
   c. Carefully construct a graph of the function showing how it passes through the points where $y = -2, y = 2, y = \pm 1$ and how it approaches the vertical asymptotes.
   ANS: $\sec(2x-1) = 2 \Leftrightarrow \cos(2x-1) = \frac{1}{2}$, if $2x - 1 = \pm\frac{\pi}{3} \Leftrightarrow x = \frac{3\pm\pi}{6} \approx -0.02$ or 1.02
   whereas $\sec(2x-1) = -2 \Leftrightarrow \cos(2x-1) = -\frac{1}{2}$ if $2x - 1 = \pi \pm \frac{\pi}{3} \Leftrightarrow x = \frac{1+\pi}{2} \pm \frac{\pi}{6} \approx 1.5$ or 2.6
   So our table of values might include

x	$\frac{3-\pi}{6} \approx -0.02$	$\frac{1}{2} = 0.5$	$\frac{3+\pi}{6} \approx 1.02$	$\frac{3+2\pi}{6} \approx 1.5$	$\frac{1+\pi}{2} \approx 2.1$	$\frac{3+4\pi}{6} \approx 2.6$
y	2	1	2	-2	-1	-2



7. Suppose $\sin t = 2/3$ and t is in the first quadrant. Find the following:
   a. $\cos(t + \pi) = -\cos(t) = -\sqrt{1 - \frac{4}{9}} = -\frac{\sqrt{5}}{3}$
   b. $\cos\left(t + \frac{\pi}{2}\right) = -\sin t = -\frac{2}{3}$
   c. $\cos\left(\frac{\pi}{2} - t\right) = \sin t = \frac{2}{3}$