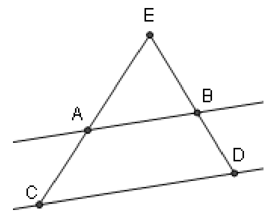
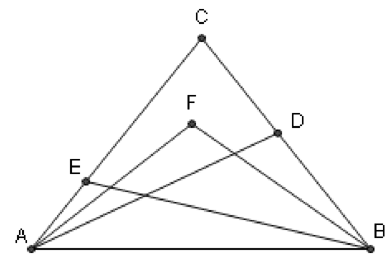
Math 5 – Trigonometry – Geometry Test – Take-home problems.
These will account for 20% of the test score. All problems should be written on separate paper and in your own writing.

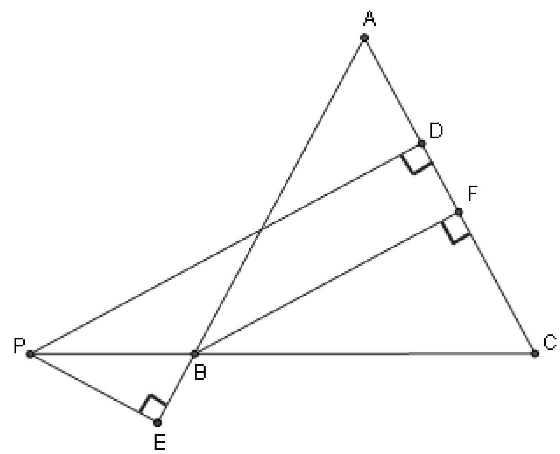


1. Prove that if $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$ then $\frac{EA}{EB}=\frac{AC}{BD}$.

2. Prove that the base angles of an isosceles trapezoid are congruent. Use the result of #1.



3. In any ΔABC, take E and D as points on the interior of segments AC and BC, respectively (see the figure.) AF bisects $\angle CAD$, and BF bisects $\angle CBE$.
   a. Prove that $\angle AEB+\angle ADB=2\angle AFB$
   b. Prove this is true even if E coincides with C.
   c. Prove that this is true even if E and D are exterior points on the extensions of AC and BC.



4. In isosceles ΔABC ($AB = AC$), CB is extended through B to P (see figure.) A line from P, parallel to altitude BF, meets AC at D, (where D is between A and F.) From P a perpendicular line is drawn to meet the extension of AB at E so that B is between E and A. Express BF in terms PD and PE.

Solution strategy: Start by looking for similar triangles. In this case, an important similarity is established by noting that, since ΔABC is isosceles, $\angle ABC\cong\angle ACB$ and vertical angles $\angle ABC\cong\angle PBE$ so right triangles ΔBFC and ΔPEB have one of their acute angles congruent and so must be equiangular (since the sum of interior angles of any triangle is 180˚.) Thus $\angle BFC\sim\angle PEB$ and among the many proportionality equations we could write, consider

(1.1) $$\frac{\text{long leg}}{\text{long leg}}=\frac{\text{hypotenuse}}{\text{hypotenuse}}\Leftrightarrow\frac{PE}{BF}=\frac{PB}{BC}.$$

Now, if a line parallel to one side of a triangle and intersects the other two sides, as BF is parallel to PD and intersects PC and DC, then the triangles are equiangular and so similar. Thus $\angle BFC\sim\angle PDC$, and again, $\frac{\text{long leg}}{\text{long leg}}=\frac{\text{hypotenuse}}{\text{hypotenuse}}\Leftrightarrow\frac{PD}{BF}=\frac{PC}{BC}=\frac{PB+BC}{BC}$. This also means that

(1.2) $$\frac{PD}{BF}-1=\frac{PC}{BC}-1\Leftrightarrow\frac{PD-BF}{BF}=\frac{PC-BC}{BC}=\frac{PB}{BC}$$

Combining 1.1 and 1.2,

(1.3) $$\frac{PD-BF}{BF}=\frac{PE}{BF}$$

Which means the numerators must be equal and so $PD-BF=PE\Leftrightarrow BF=PD-PE$. QED

Here's an alternate proof, in the form of a sequence of problems:

Justify each of the following statements (refer to the statement of the problem preceding the first proof above.)

a. *PD* is parallel to *BF*.
b. $\angle ACB \cong \angle PBE$
c. $\angle EPB \cong \angle FBC$
d. $\angle FBC \cong \angle DPC$
e. Draw a line from *B* perpendicular to *PD* at *G*. Then $\angle GPB \cong \angle EPB$.
f. $\Delta GPB \cong \Delta EPB$
g. $BF=GD$
h. $BF = PD - PE$

5. Research Euclid's proof for the Pythagorean theorem and write it up in your own words.

6. Find a proof for the Pythagorean theorem which is different from Euclid's and different from the Chou Pei Suan Ching version discussed in class.