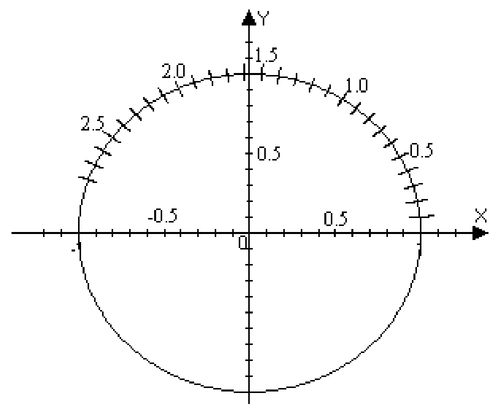
Math 5 – Trigonometry – Chapter 4 Test – fall ’06   Name___________________________
Show your work for credit. Write all responses on separate paper.

1. For arclength $t = \dfrac{17\pi}{6}$ extending counterclockwise along the unit circle from (1,0)
   a. Find the reference number for *t*.
   b. Find the coordinates of the terminal point *P*(*x*,*y*).
   c. Illustrate this point’s position on a plot of the unit circle.

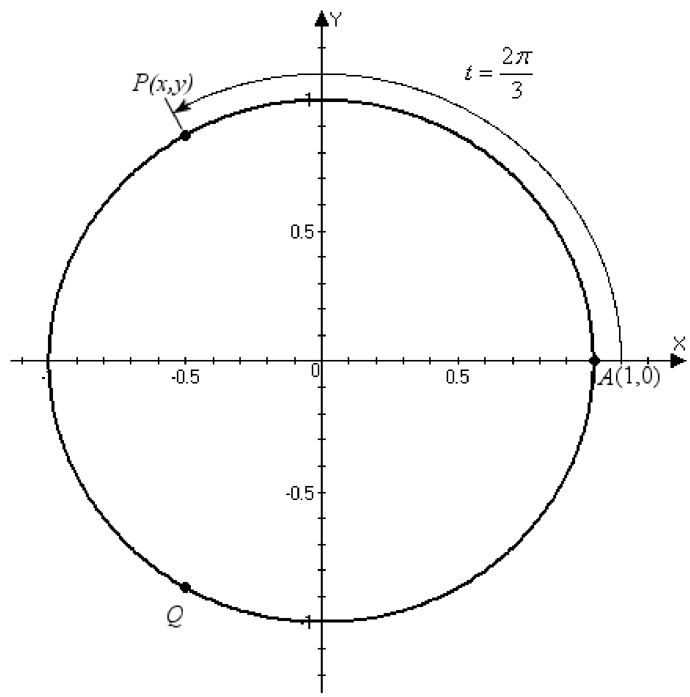
2. Suppose $0.9 \le t \le 1.9$ highlight that interval on the perimeter of the unit circle



   a. Approximate to the nearest tenth the interval of *x* values corresponding to this *t* interval.

   b. Approximate to the nearest tenth the interval of *y* values corresponding to this *t* interval.

3. In the diagram at right, *P* is the terminal point for $t = \dfrac{2\pi}{3}$ and *Q* is the reflection of *P* across the *x*-axis.



   a. Explain why ΔAPQ is equilateral.

   b. What are the coordinates of *Q* in terms of *x* and *y*?

   c. Set up an equation in terms of *x* and *y* based on $\overline{AP}^2 = \overline{PQ}^2$ and show this is equivalent to $3y^2 = (x-1)^2$.

   d. Use the fact that (*x*,*y*) is on the unit circle to substitute for *y* in terms of *x* and solve the resulting equation for *x*.

   e. Substitute your value for *x* into the equation for in (c) and solve for *y*.

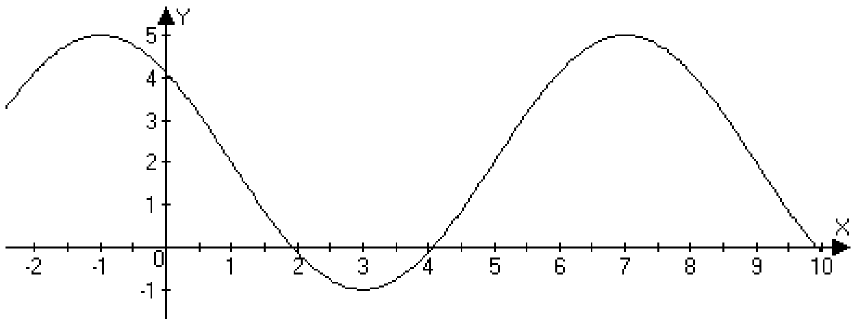
4. The terminal point for an arc length *t* on the unit circle is $\left(\dfrac{16}{65}, -\dfrac{63}{65}\right)$.

   Find *sin t*, *cos t* and *tan t*.

5. Write *tan t* in terms of *cos t*, assuming the terminal point for *t* is in quadrant II.

6. Find the amplitude, period and phase shift of $y = -3 + 2\sin\left(8\pi\left(x + \frac{1}{16}\right)\right)$ and construct a careful, large graph showing one period of the function.

7. Find an equation for the sinusoid whose graph is shown:



8. Consider the function $f(x) = \tan\left(\frac{1}{2}\left(x + \frac{\pi}{4}\right)\right)$.
   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
   b. Find the *x*-coordinates of three points that divide the interval between the vertical asymptotes into 4 equal parts and evaluate the function at these three points.
   c. Construct a careful, large graph of the function showing how it passes through the three points and how it approaches the vertical asymptotes.

9. Suppose *sin t* = 3/5 and *t* is in the first quadrant. Find the following:
   a. $\sin(t + \pi)$
   b. $\sin\left(t + \frac{\pi}{2}\right)$
   c. $\sin\left(\frac{\pi}{2} - t\right)$

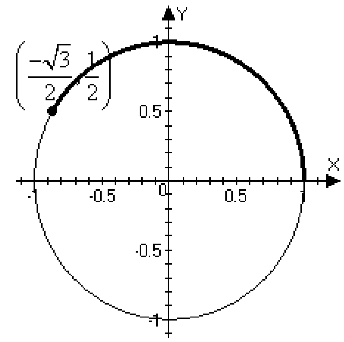
Trigonometry – Chapter 4 Test Solutions – Fall '06

1. For arclength $t = \dfrac{17\pi}{6}$ extending counterclockwise along the unit circle from (1,0)

   a. The reference number for t is $\bar{t} = \left|\dfrac{17\pi}{6} - 3\pi\right| = \dfrac{\pi}{6}$

   b. The coordinates of the terminal point $P(x,y)$ are then $\left(\dfrac{-\sqrt{3}}{2}, \dfrac{1}{2}\right)$:

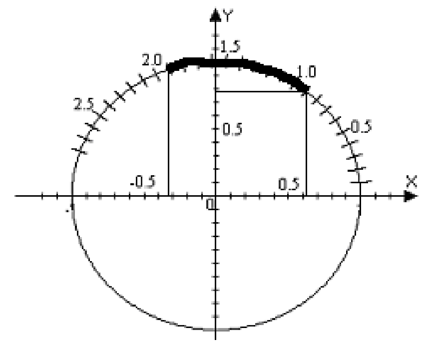
   c. The diagram at right illustrates this point's position on a plot of the unit circle.



2. Suppose $0.9 \le t \le 1.9$ highlight that interval on the perimeter of the unit circle

   a. If $0.9 \le t \le 1.9$, then, as shown in the diagram, $0.3 \le x \le 0.6$ This is equivalent to $0.3 \le \cos t \le 0.6$.

   b. If $0.9 \le t \le 1.9$, then, as shown in the diagram, $0.8 \le y \le 1$ This is equivalent to $0.8 \le \sin t \le 1$

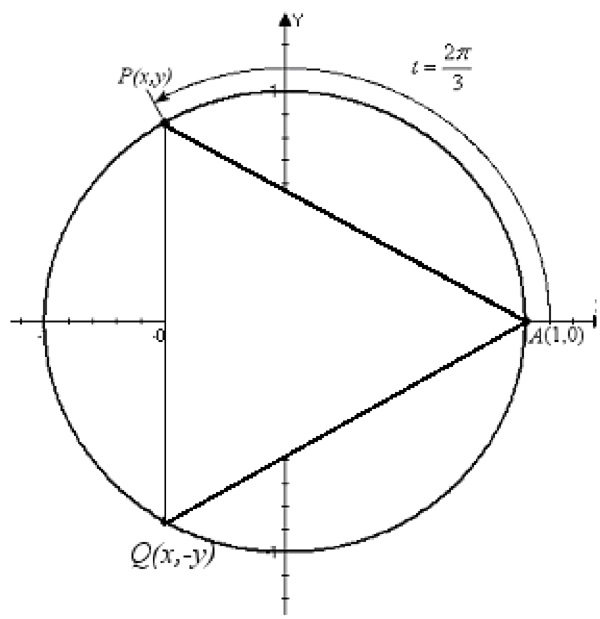


3. In the diagram, P is the terminal point for $t = \dfrac{2\pi}{3}$ and Q is the reflection of P across the x-axis.

   a. Since the directed arc from A to Q, $t = -\dfrac{2\pi}{3}$ has the same length: $\widehat{AP} = \widehat{AQ}$. But this means that the arc $\widehat{QP} = 2\pi - \widehat{AP} - \widehat{AQ} = \dfrac{2\pi}{3}$ so that $\widehat{AP} = \widehat{AQ} = \widehat{PQ}$ whence the corresponding chords are also equal and ΔAPQ is equilateral.

   b. The coordinates of Q are $(x,-y)$.

   c. $\overline{AP}^2 = \overline{PQ}^2 \Leftrightarrow (2y)^2 = (1-x)^2 + y^2$
   $\Leftrightarrow 3y^2 = (x-1)^2$



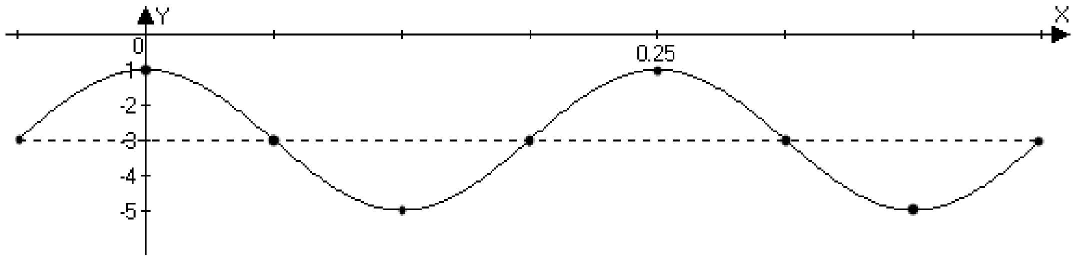
   d. Since (x,y) is on the unit circle, $y^2 = 1 - x^2$ and we can to substitute for y to get $3(1-x^2) = (x-1)^2$. Expand, collect and get zero on one side:
   $\Leftrightarrow 3 - 3x^2 = x^2 - 2x + 1 \Leftrightarrow 2x^2 - x - 1 = 0$, which is factorable:
   $(2x+1)(x-1) = 0 \Rightarrow x = -1/2$ or $x = 1$

   e. If $x = -1/2$ then $y = \pm\sqrt{1 - \left(-\dfrac{1}{2}\right)^2} = \pm\sqrt{\dfrac{3}{4}} = \pm\dfrac{\sqrt{3}}{2}$.

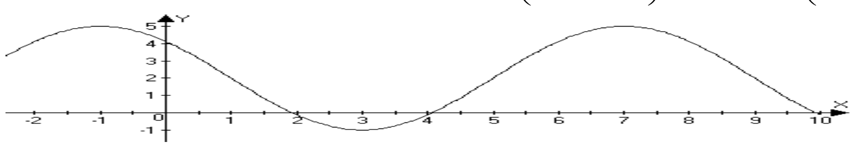
4. $\left(\dfrac{16}{65}, -\dfrac{63}{65}\right)$ implies $\sin t = -\dfrac{63}{65}$, $\cos t = \dfrac{16}{65}$ and $\tan t = -\dfrac{63}{16}$

5. In quadrant II $\tan t = \dfrac{\sin t}{\cos t} = \dfrac{\sqrt{1-\cos^2 t}}{\cos t}$

6. The amplitude, period and phase shift of $y = -3 + 2\sin\left(8\pi\left(x + \dfrac{1}{16}\right)\right)$ is 2, the period is ¼ and the phase shift is -1/16. Here's a graph showing two periods.



7. The sinusoid shown below has *y* values ranging from -1 to 5 so the line of equilibrium is *y* = 2 and the amplitude is 3. There's a peak at *x* = -1 and the next is at *x* = 7, so the period is 8. The wave is at equilibrium at *x* = 5 so the phase shift is -3. Thus the sinusoid can be expressed either as a sine or a cosine: $y = 2 + 3\sin\left(\dfrac{\pi}{4}(x+3)\right) = 2 + 3\cos\left(\dfrac{\pi}{4}(x+1)\right)$



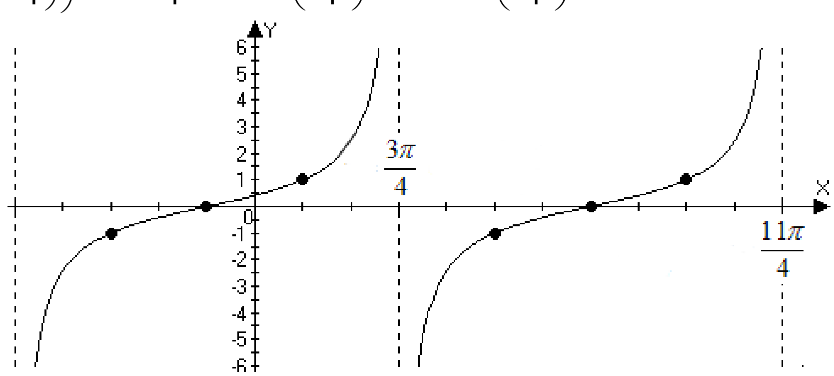
8. Consider the function $f(x) = \tan\left(\dfrac{1}{2}\left(x + \dfrac{\pi}{4}\right)\right)$.

   a. Vertical asymptotes can be found by solving $\dfrac{1}{2}\left(x + \dfrac{\pi}{4}\right) = \dfrac{\pi}{2} + k\pi$ where *k* is any integer:

   $\dfrac{1}{2}\left(x + \dfrac{\pi}{4}\right) = \dfrac{\pi}{2} + k\pi \Leftrightarrow x + \dfrac{\pi}{4} = \pi + 2\pi k \Leftrightarrow \boxed{x = \dfrac{3\pi}{4} + 2\pi k}$, whence adjacent asymptotes can be found by plugging in, say, *k* = 0 and *k* = 1: $x = \dfrac{3\pi}{4}$ or $x = \dfrac{11\pi}{4}$

   b. The distance between asymptotes is one wavelength, so we add a quarter wavelength to the first asymptote three times to get $x_1 = \dfrac{3\pi}{4} + \dfrac{2\pi}{4} = \dfrac{5\pi}{4}$, $x_2 = \dfrac{7\pi}{4}$, $x_3 = \dfrac{9\pi}{4}$. Thus the points to plot are $f\left(\dfrac{5\pi}{4}\right) = \tan\left(\dfrac{1}{2}\left(\dfrac{5\pi}{4} + \dfrac{\pi}{4}\right)\right) = \tan\dfrac{3\pi}{4} = -1$, $f\left(\dfrac{7\pi}{4}\right) = 0$ and $f\left(\dfrac{9\pi}{4}\right) = 1$

   c. The graph at right shows how the function passes through the three points and approaches the vertical asymptotes.



9. Suppose *sin t* = 3/5 and *t* is in the first quadrant. Find the following:

   a. $\sin(t+\pi) = -\sin t = -\dfrac{3}{5}$ b. $\sin\left(t + \dfrac{\pi}{2}\right) = -\cos t = -\dfrac{4}{5}$ c. $\sin\left(\dfrac{\pi}{2} - t\right) = \cos t = \dfrac{4}{5}$