Math 5 – Trigonometry – spring '09 – Chapter 3 Test   Name____________________

Show all work for credit. With the exceptions of #6 and #7 write all responses on separate paper.

1. Consider the line that intersects $f(x) = (x-2)^2 + 1$ where $x = 2$ and where $x = 4$.
   a. Find the slope of the line.
   b. Find an equation for the line.
   c. Construct a careful graph the line and the parabola together showing the points of intersection, and the parabola's vertex and all intercepts.
   d. Find an equation for the line with slope = 2 that intersects the parabola only once. *Hint*: In the equation $y = mx + b$ Substitute for the y coordinate on the parabola in terms of x and substitute the slope for m and then choose b so the discrimininant of the quadratic equation is zero.

2. For each of the functions below, the domain is all real numbers. Express the range of each function using interval notation. Tabulate at least 5 input/output pairs and construct a careful graph showing how the function passes through these points.
   a. $f(x) = 2x^2 - 8x + 9$
   b. $g(x) = \dfrac{1}{2(x-2)^2 + 1}$

3. Consider the square root function, $f(x) = 2 - \sqrt{x-1}$
   a. Write the domain and range of this function using interval notation. Note: restrict the domain so that the output is a real number.
   b. Make a table of values for the function and sketch its graph showing the intercepts and at least two other points.
   c. What is the domain of $(f \circ f)(x)$? *Hint*: be sure that the output of *f* is in the domain of *f*.

4. Suppose the graph of *f* is given. Describe how the graph of each function can be obtained from the graph of *f* in terms of horizontal and/or vertical shifts, shrinking and/or stretching and reflections.
   a. $y = f(x-2) + \sqrt{2}$
   b. $y = 1 - \dfrac{1}{2} f(2x)$

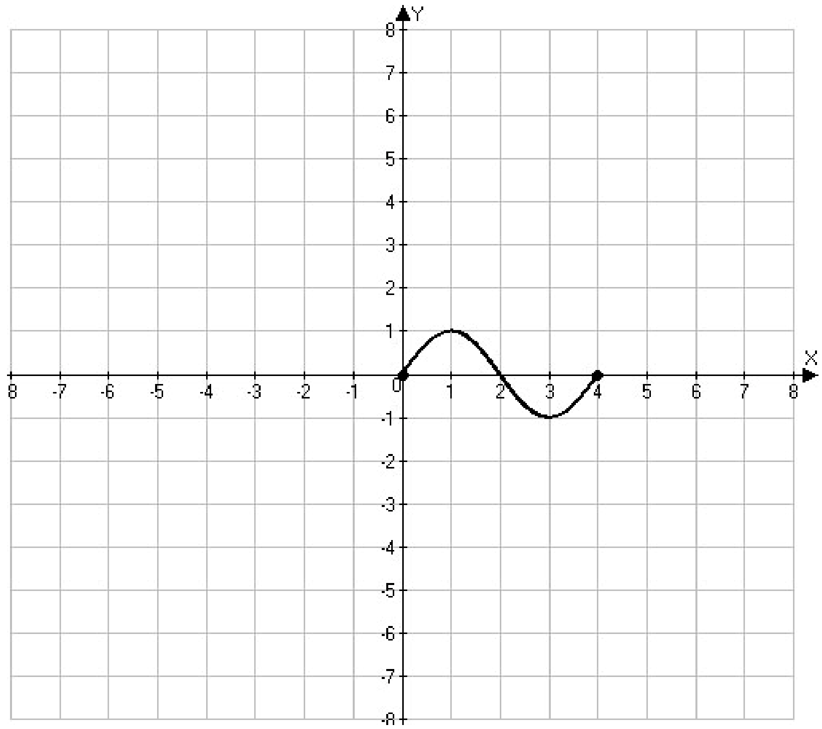
5. Suppose $f(x) = \dfrac{1}{\sqrt{x}}$ and $g(x) = x^2 - 4$. Find the following and state their domains:
   a. $f \circ g$
   b. $g \circ f$
   c. $f \circ f$
   d. $g \circ g$

6. Given the graph of $y = f(x)$ shown at right, graph and label the following transformations in the space provided.

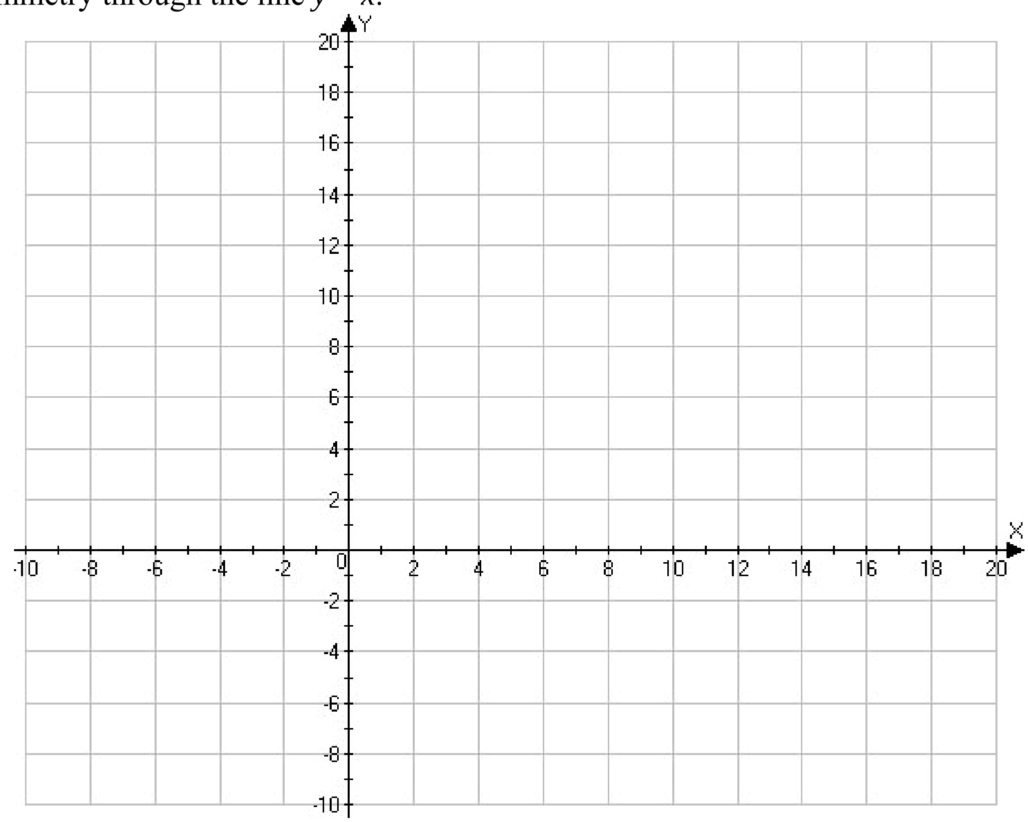
   a. $y_1 = f(x+4)$

   b. $y_3 = f(-x) - 2$

   c. $y_2 = 1 - 2f\left(\frac{x}{2}\right)$



7. Consider the function $f(x) = 8 + 4\sqrt[3]{x-8}$
   a. Find an inverse function formula for *f*. *Hint*: This is a cubic polynomial formula.
   b. Tabulate (x, y) pairs for $y = f(x)$ for $x = 0$, $x = 7$, $x = 8$, $x = 9$, and $x = 16$.
   c. Use this table to sketch graphs for $f^{-1}(x)$ and $f(x)$ together showing the symmetry through the line $y = x$.



Math 5 – Trigonometry – Spring '09 – Chapter 3 Test Solutions

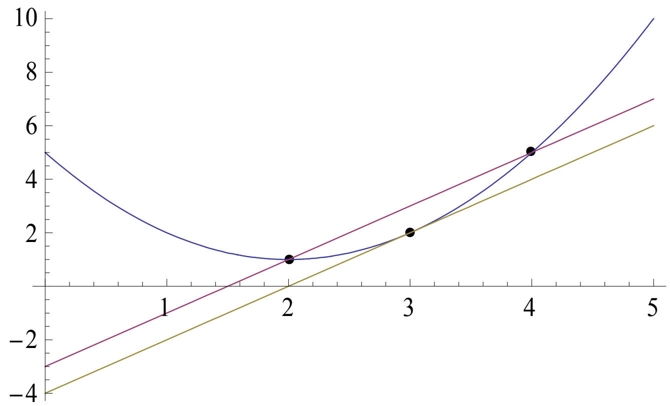
1. Consider the line that intersects intersects $f(x) = (x-2)^2 + 1$ where $x = 2$ and where $x = 4$.

   a. Find the slope of the line. SOLN: $\frac{f(4) - f(2)}{4-2} = \frac{(4-2)^2 + 1 - ((2-2)^2 + 1)}{2} = \frac{5-1}{2} = 2$

   b. Find an equation for the line. SOLN: Using the point slope formula, $y - 1 = 2(x-2) \Leftrightarrow y = 2x - 3$

   c. Construct a careful graph the line and the parabola together showing the points of intersection, and the parabola's vertex and all intercepts.

   d. Find an equation for the line with slope = 2 that intersects the parabola only once.



A line with slope 2 will have the equation $y = 2x + b$. Equating this y value with the y value on the parabola yields $2x + b = (x-2)^2 + 1 \Leftrightarrow x^2 - 6x + 5 - b = 0$. If this equation is to have only one solution then the discriminant must be zero: $36 - 4(1)(5 - b) = 0$ or $b = -4$
Thus the line is $y = 2x - 4$.

2. For each of the functions below, the domain is all real numbers.
   Express the range of each function using interval notation.
   Tabulate at least 5 input/output pairs and construct a careful graph showing how the function passes through these points.

   a. $f(x) = 2x^2 - 8x + 9 = 2(x-2)^2 + 1$

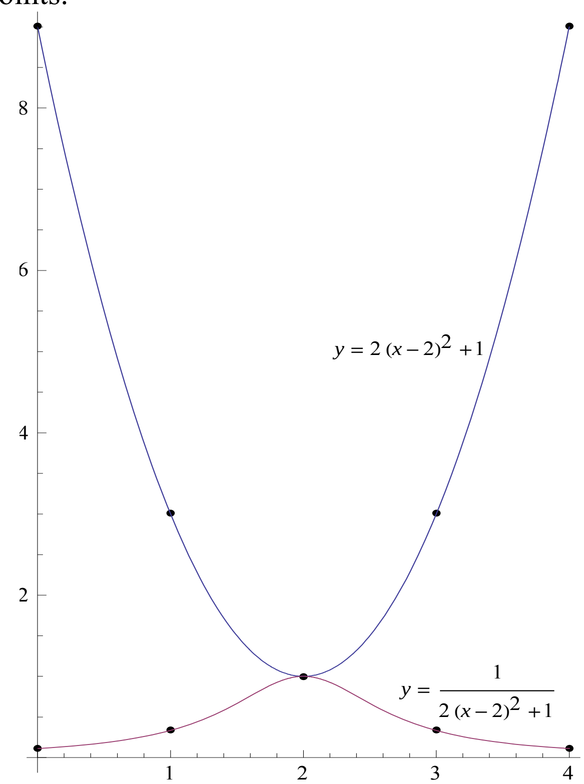
x	0	1	2	3	4
y	9	3	1	3	9

   This is a parabola opening upwards from a vertex at (2,1). The range of this function is $y \in [1, \infty)$

   b. $g(x) = \frac{1}{2(x-2)^2 + 1}$

x	0	1	2	3	4
y	$\frac{1}{9}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{9}$

This is the reciprocal of the $y \geq 1$ value in part (a) and thus the maximum is 1 and the range is $y \in (0, 1]$. The reciprocals become arbitrarily small as $|x - 2| \to \infty$ but must be greater than zero. Since the graphs are related they are shown together at right.



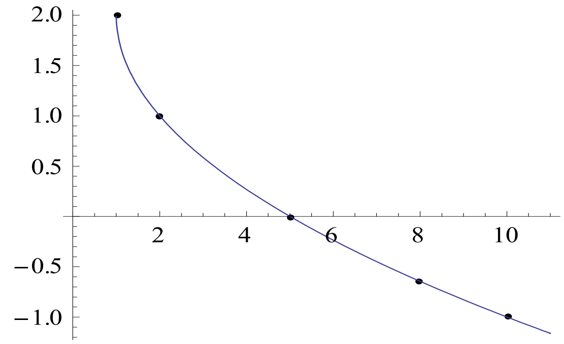
3. Consider the square root function, $f(x) = 2 - \sqrt{x-1}$

a. Write the domain and range of this function using interval notation.
Note: restrict the domain so that the output is a real number.
SOLN: For the domain we require the outputs are real valued so that $x - 1 \geq 0$ so that $x \in [1, \infty)$ is required for x to be in the domain. For the range, observe that $\sqrt{x-1} \geq 0 \Leftrightarrow -\sqrt{x-1} \leq 0 \Leftrightarrow 2 - \sqrt{x-1} \leq 2$ so the range is $(-\infty, 2]$

b. Make a table of values for the function and sketch its graph showing the intercepts and at least two other points.

SOLN:

x	1	2	5	8	10
y	2	1	0	$2-\sqrt{7}$	-1



This is the lower half a parabola whose vertex equation is $x = (y-2)^2 + 1$

c. What is the domain of $(f \circ f)(x)$? *Hint*: be sure that the output of *f* is in the domain of *f*.
SOLN: The output of *f* is in the interval $(-\infty, 2]$, we want to require that it is also in the domain, $x \in [1, \infty)$. Thus the domain is [1,2]. To be sure look at the formula for the composition, $f(f(x)) = f\left(1 - \sqrt{x-1}\right) = 2 - \sqrt{\left(2 - \sqrt{x-1}\right) - 1} = 2 - \sqrt{1 - \sqrt{x-1}}$ which confirms the conclusion.

4. Suppose the graph of *f* is given. Describe how the graph of each function can be obtained from the graph of *f* in terms of horizontal and/or vertical shifts, shrinking and/or stretching and reflections.

a. $y = f(x-2) + \sqrt{2}$
SOLN: Shift right 2 and up $\sqrt{2}$

b. $y = 1 - \frac{1}{2} f(2x)$
Reflect across *x*-axis, shrink horizontally by 2, shrink vertically by 2 and shift up 1.

5. Suppose $f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = x^2 - 4$. Find the following and state their domains:

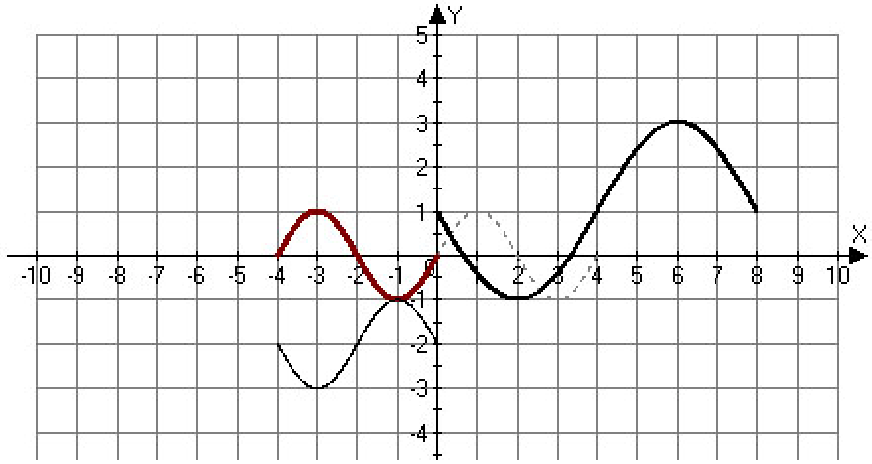
a. $f \circ g$ SOLN: $f(g(x)) = \frac{1}{\sqrt{x^2 - 4}}$ has domain $(-\infty, -2) \cup (2, \infty)$

b. $g \circ f$ SOLN: $g(f(x)) = \left(\frac{1}{\sqrt{x}}\right)^2 - 4$ has domain $(0, \infty)$.

c. $f \circ f$ SOLN: $f(f(x)) = f\left(x^{-1/2}\right) = \left(x^{-1/2}\right)^{-1/2} = x^{1/4} = \sqrt{\sqrt{x}}$, has domain $(0, \infty)$

d. $g \circ g$ SOLN: $g(g(x)) = \left(x^2 - 4\right)^2 - 4$ has all real numbers as the domain.

6. Given the graph of $y = f(x)$ shown at right, graph and label the following transformations in the space provided.



   a. $y_1 = f(x+4)$
      SOLN: Shift 4 left.
   b. $y_3 = f(-x)-2$
      SOLN: Reflect in *y* axis and shift 2 down
   c. $y_2 = 1-2f\left(\frac{x}{2}\right)$ Stretch both horizontally and vertically by 2, reflect in *x*-axis and shift 1 up.

7. Consider the function $f(x) = 8+4\sqrt[3]{x-8}$
   a. Find an inverse function formula for *f*.

      SOLN: $y = 8+4\sqrt[3]{x-8} \Leftrightarrow \sqrt[3]{x-8} = \frac{y-8}{4} \Leftrightarrow x-8 = \left(\frac{y-8}{4}\right)^3 \Leftrightarrow f^{-1}(x) = 8+\frac{(x-8)^3}{64}$
   b. Tabulate (*x*, *y*) pairs for $y = f(x)$ for *x* = 0, *x* = 7, *x* = 8, *x* = 9, and *x* = 16.

x	0	7	8	9	16
$f(x)$	0	4	8	12	16

   Note: simply swap *x* and *y* values for the inverse table.

   c. Use this table to sketch graphs for $f^{-1}(x)$ and $f(x)$ together showing the symmetry through the line $y = x$.