

Consider the following precondition and postcondition specifications:

**Precondition** A list of  $n$  different integers:  $\{x_1, x_2, \dots, x_n\}$  so that  $x_i \neq x_j$  if  $i \neq j$

**Postcondition** The same list of numbers sorted in increasing order.

For example, the list  $\{2, 1, 5, 3\}$  could be an input (precondition) and then the postcondition would be  $\{1, 2, 3, 5\}$ .

1. Write pseudocode for an algorithm that will accomplish this.

Did I say “sort”? Oh dear...that’s potentially complicated! I had a simpler notion in mind, but the sort thing just kinda popped out! Oh, hell...so here it goes:

There are many, many sort routines. Most computer time is spent searching and sorting; sorting makes it easier to search...

Here’s a link to a fairly comprehensive collection of sorting algorithms: <https://www.toptal.com/developers/sorting-algorithms>. Let’s pick selection sort, whose algorithm in pseudo code might look like this:

```
for i = 2:n,
  for (k = i; k > 1 and a[k] < a[k-1]; k--)
    swap a[k,k-1]
  invariant: a[1..i] is sorted
end
```

Ok, I take it back. Let’s do bubblesort. The aforementioned sorting web site has this algo:

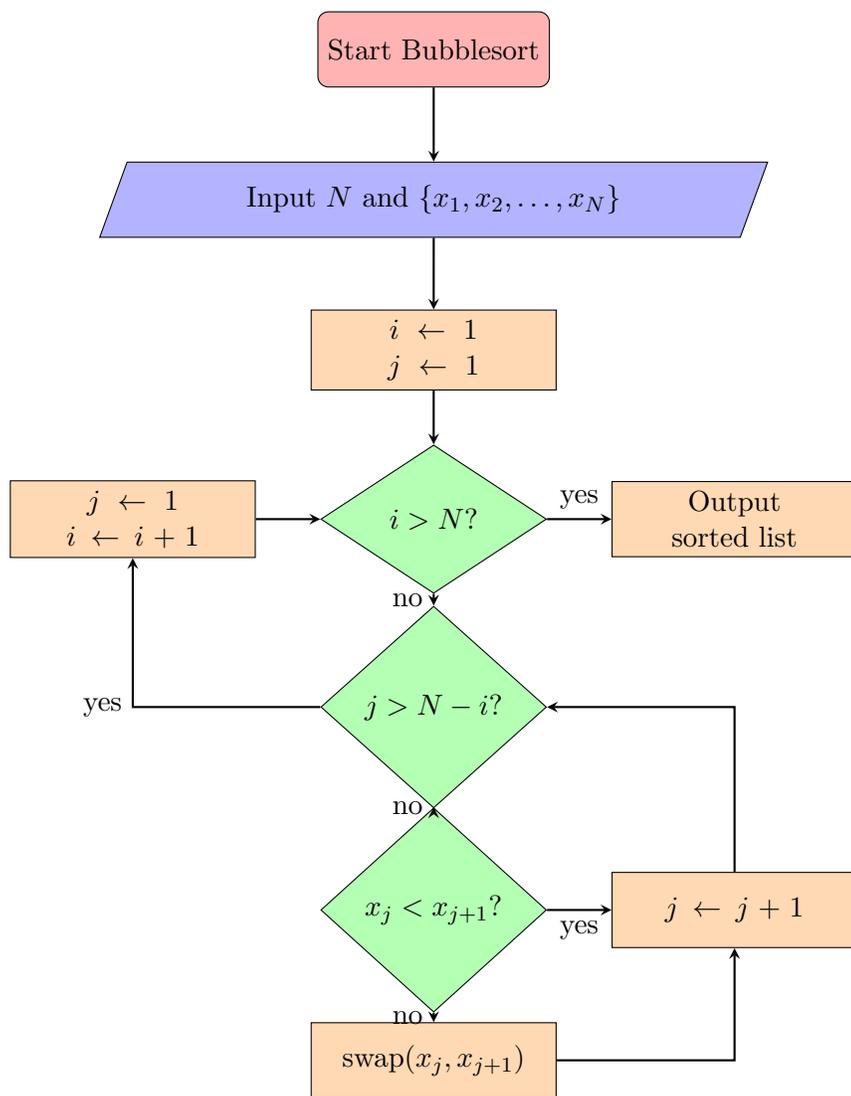
```
for i = 1:n,
  swapped = false
  for j = n:i+1,
    if a[j] < a[j-1],
      swap a[j,j-1]
      swapped = true
  invariant: a[1..i] in final position
  break if not swapped
end
```

That’s no doubt improved over mine:

```
for i = 1 to N,
  for j = 1 to N-i
    if x[j]>x[j+1]
      swap(x[j],x[j+1])
```

2. Sketch a flow-chart for your algorithm.

Here you go:



3. Write C++ code that will implement your algorithm.

```

1 #include <iostream>
2 #include <vector>
3 #include <cstdlib>
4 #include <ctime>
5
6 using namespace std;
7
8 /**
9  for i = 1 to N,
10     for j = 1 to N-i
11         if x[j]>x[j+1]
12             swap(x[j],x[j+1])
13 */
14
15 int bubbleSort(vector<int>& v) {
16     int swaps{0};
17     for(int i = 0; i < v.size(); ++i)
18         for(int j = 0; j < v.size()-i-1; ++j)
19             if(v[j]>v[j+1]) {
20                 swap(v[j],v[j+1]);
21                 ++swaps;
22             }
23     return swaps;
24 }
25
26 void show(vector<int>& v) {
27     for(int i:v) cout << i << " ";
28 }

```

```

29 void initializeV(int N, vector<int>& v, bool randm = 1) {
31     if(randm) {
33         for(int i = 0; i < v.size(); ++i) v[i]=(rand()%10000);
35     }
37     else for(int i = 0; i < N; ++i) {
39         cout << "\nEnter c[" << i << "] = ";
41         cin >> v[i];
43     }
45 }
47
49 int main() {
51     int N{100};
53     vector<int> v(N);
55     int sumSwaps{0};
57     for(int n = 0; n < 1000; ++n) {
59         initializeV(N,v);
61         sumSwaps += bubbleSort(v);
63     }
65     cout << "\nThe percentage of swaps = " << double(sumSwaps)/(1000*N);
67     return 0;
69 }

```

4. What is an invariant in your code?

After iteration  $i$  the values  $\{x_{N-i}, x_{N-i+1}, \dots, x_N\}$  will contain the  $i$  largest values of the list in ascending order.

5. What is the number of comparisons your code requires for a list of  $n$  different integers?

The  $i$ th pass through the outer loop requires  $N-i$  comparisons, so there are a total of  $N+(N-1)+(N-2)+\dots+1 = \frac{N(N-1)}{2}$  comparisons.

6. What is the average number of times your algorithm will swap two numbers for a list of three integers?

The code above gives a general empirical result of about 24%.