

## CS007A: Sam Lloyd's Fifteen Puzzle - Part 2

---



According to the Wolfram Mathworld page at <http://mathworld.wolfram.com/15Puzzle.html>, you can determine whether an arrangement of the 15 puzzle is solvable by looking at the sum formed by counting the number of numbers to the right and below each number in the arrangement that is smaller than that number (therefore out of order.) The puzzle is solvable only if this number is even.

For example, in the arrangement at right, the counts are 12, 9, 9, 5, 4, 4, 3, 3, 0, 3, 3, 2, 1, 1, and 0. That is there are 12 numbers beyond the 13 that are less than 13, 9 numbers beyond the 10 that are less than 10, 9 number beyond the 11 that are less than 11, and so on. In this case, the sum is 59, which is odd, and so the arrangement is unsolvable.

13	10	11	6
5	7	4	8
1	12	14	9
3	15	2	

Whoops. Looks like trust in Wolfram's Mathworld has been misplaced here! Computing the inversion number and observing its change after an up or down move, it becomes obvious that it is changing from odd to even or even to odd...meaning that the polarity of the inversion does not determine the solvability! Oops!

For example, suppose we move the 9 down into the blank in the above arrangement. This move will subtract 2 from the inversion because 3 and 2 are no longer after 9, and will add 1 to the inversion because now 9 is after 15. The total change then is to subtract 1 from the inversion, changing it from the odd value 59, to the even value 58 – but that certainly doesn't change the fact of whether or not the puzzle is solvable!

It turns out the situation is more nuanced when the square has an even number of rows and columns (4 in this case.) Mark Ryan, at the University of Birmingham in England, has a web site that not only gives a better formula for solvability, but proves it. See <http://www.cs.bham.ac.uk/~mdr/teaching/modules04/java2/TilesSolvability.html>

Ryan's formula says:

- a. If the grid width is odd, then the number of inversions in a solvable situation is even.
- b. If the grid width is even, and the blank is on an even row counting from the top, then the number of inversions in a solvable situation is even.
- c. If the grid width is even, and the blank is on an odd row counting from the top then the number of inversions in a solvable situation is odd.

So the grid we have in our example with inversion value 59 has the blank in an even row (4<sup>th</sup> row) so it the puzzle is not solvable.

Your task is to write a function called `inversion` that takes an arrangement array and computes the inversion sum (59, in the above case.) Use this to make sure that a given arrangement is solvable before displaying. Be sure to check whether the blank is in an odd or even row.

Further, modify the program to take the user out of the play and have the computer compute the inversion number for all possible moves and then make the move that minimizes the inversion number.