MathFest '14 Report

Geoff Hagopian

August 26, 2014

I arrived at the conference somewhat unprepared. The week immediately prior to the meeting I'd been in Montana at places completely off the grid: no internet – no telephone! I'd planned this, but hadn't included prep for MathFest in my plans, other than to register and log in for the minicourse in SIMIODE.

First Person Solvers – Using Video Games to Learn Mathematics and Solve Real Math Problems

This was the first talk I attended, an invited address in the Main Ballroom. I hadn't looked at the name of the presenter, and wondered at the faintly familiar dulcet London cooing until I recognized it: Keith Devlin, NPR's Math Guy. Ok.

He was making an analogy about how math writing became less narrative and more symbol-oriented with the advent of the printing press in the same way that math will become more interactive with the advent of the computer tablet. "Where is the music?" he asked, showing a picture of music notation juxtaposed with a picture of someone playing the piano. Then, math symbolism with a picture of the brain. He describes this as a symbol barrier to be overcome. This seemed a stretch, but...it's NPR's Math Guy, so what do you expect?

Devlin is up-front about his disappointment when NSF rejected his proposals for developing math game software and how happy he was to have venture capitalists jump in to support him instead, and his BrainQuake project. He gave an overview of some math game/ed software that is pretty good, these two, for example: BigBrainz Math Attack, Ko's Journey. Math Attack is sort of first person shooter where, instead of a gun to shoot at a target, you get a computation to perform, like 9*12. Ko's Journey involves an "ancient text" a la Myst and that sort of thing.

BrainQuake's produces Wuzzit Trouble, which runs on the IPhone (and maybe some kind of Google cloud thing?) Give it a whirl. I've spent about a 1/2 hour so far and it offers very simple Diophantine problems at its most challenging...so far. Here are some of the evaluations at their site:

> Excellent Mathematical Game For Every Age The game trains you to do mental arithmetic involving multiplication, addition and subtraction over a base-65 system. The idea is given a set of cogs, attempt to find an integer partition involving the cogs and certain arithmetic combinations so that the partition hits the keys, gets the goodies, avoid the baddies, such that the number of steps are within the maximum allowed to attain 3-star (highest rating). The description sounds complicated but the beauty of it is such that once you start playing the game, it is really

> natural and comes together easily. You end up learning modulo arithmetic, integer decompositions, and optimization while playing a cute game.

and

> Sorry, Keith I have tried it, and I have asked others to try it. From kids age 10 to adults over 50. The game was found tedious, dull and "Booooring". Somehow, the educational aspect is too thick and the game is too thin.

I think Devlin doth protest a tad too much and that he's behaving a bit more like a venture capitalist mouthpiece than a careful student of math ed...but maybe I'm too much in the box to receive this enlightenment. I will keep my mind open.

Here are some other math computer games to consider:

- Dragonbox Algebra
- Motion Math
- Kickbox
- Refraction (free UW game).

After describing this to a gamer/developer I was advised to check out this parody: Frog Fractions – har!

I missed Ricardo Cortez' talk on Understanding Microorganisms Swimming, but you can follow the link to his paper, which looks quite diffy Q.

Geometry Expressions

Instead I walked the exhibitions. There I found Saltire Software's President Phillip Todd presenting their product, Geometry Expressions, and their curriculum, which is, they say (and I hope to agree!) "Common Core Ready." He was very generous: gave me a personal walk through of his software and let me pick two of the various textbooks that are written involving GX software.

Developing Geometry Proofs with Geometry Expressions by: Irina Lyublinskaya, Valeriy Ryzhik, Dan Funsch has a very thoughtful introduction, showing a good understanding of geometry's precarious position in education. The introductionm poses the question, "Why is [teaching geometric proofs] so hard?" For which it offers these answers:

- Lack of motivation–students may question the necessity for proving visually obvious properties.
- Lack of an algorithm–many geometric problems cannot be reduced to calculations. Each problem is different. There is no general approach to their solutions.
- Difficulty in choosing a method of proof–solving geometry problems requires a broad knowledge of different mathematical areas (algebra, trigonometry, functions) as well as skill in knowing how to choose a method of proof.

- Where to start–when faced with a problem students can often feel helpless, not knowing where to start.

- Multi-step planning–many geometry problems require students to identify required information that is not provided. Students must recognize this and take steps to acquire the information before embarking on the main problem itself.

- Additional constructions–some geoemtry problems have indirect or "hidden" information that is not explicitly stated in the problem statement. It may require additional construction(s).

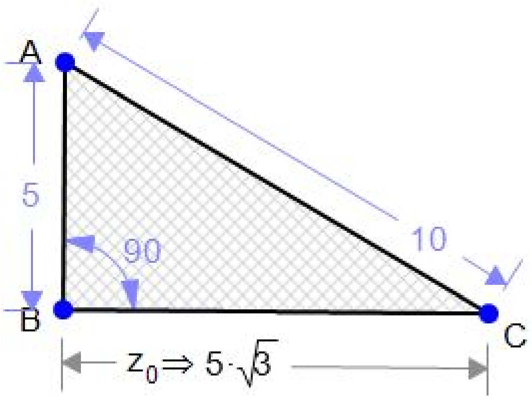
- Finding the conjecture–in some cases students may have a difficulty formulating the conjecture that is to be proved.

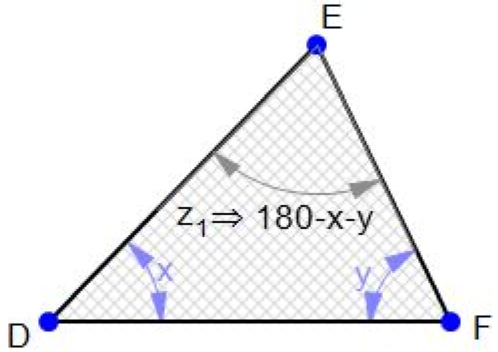
- Problem interpretation–students may not understand the problem as a whole. More specifically, they may not understand the origin of the problem, why the given information is provided, why the results are important, or why the described situation is possible at all.

From the text:

> Geometry Expressions is a dynamic, constraint-based symbolic geometry system which allows the algebraic and the diagrammatic representations of geometry to co-exist in an environment that fosters a holistic view of mathematics. It allows students to model geometric relations algebraically and to derive the results in an algebraic system. This gives students access to a powerful toolkit for participating in an inductive, exploration-based learning environment. This technology moves beyond the first generation of dynamic geometry systems, integrating geometric and algebraic explorations. From a a pedagogic standpoint an interactive symbolic geometry system affords a remarkable opportunity to make concrete the concept of a variable in readily identifiable real-world settings. Geometry Expressions provides a mathematical tool that derives results as algebraic expressions involving the input parameters. Ultimately, unlike the typical segregated curricular approach for learning mathematics, GX supports a complete mathematical system–geometry and algebra together as opposed to separate algebra, geometry and then more algebra.

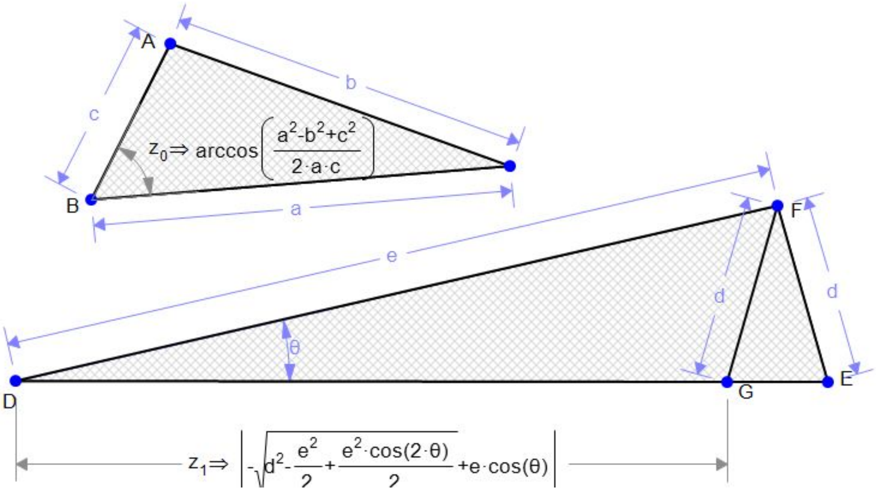
The authors claim that a constraint-based system is intrinsically different from construction-based system like Geometer's SketchPad. To see how that works, I recommend their *Using a Constraint Approach in Teaching Mathematics* available at their web site. You can constrain various geometric quantities until some other quantity(s) is determined and then ask for these to computed, using parameters or numbers intelligently in nice adjustable diagrams. To make this one I first switched to degree mode.





Maybe this is moving closer to the symbol-breaking technology that Devlin seeks? We should introduce Devling to Dr. Todd.

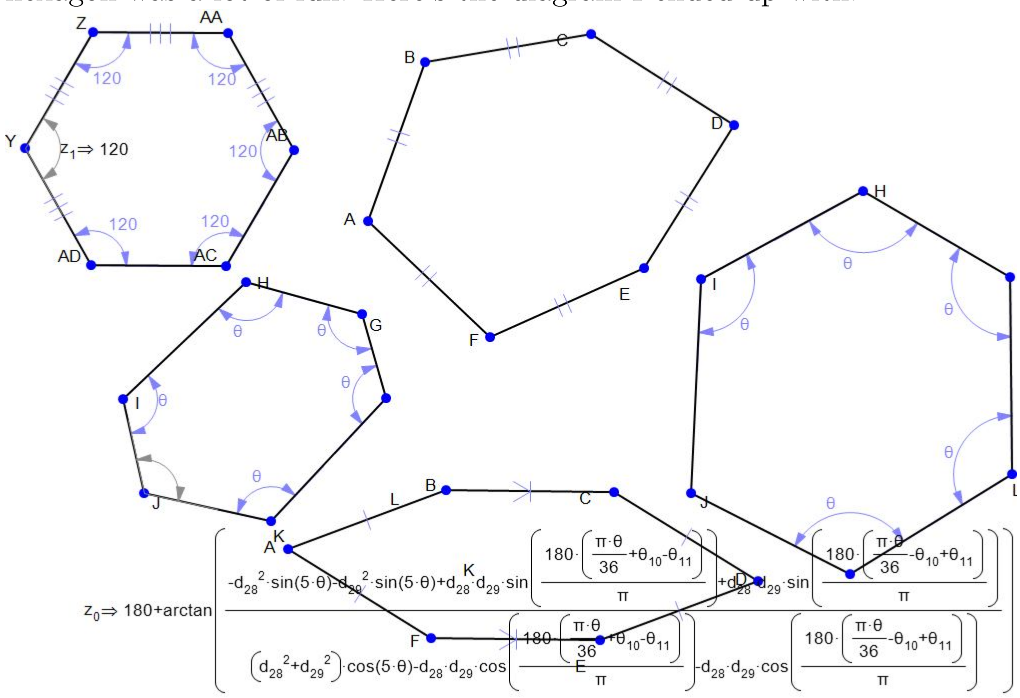
Next I looked at the law of cosines and the ASS to produce these diagrams:



When you to try to constrain a length or angle that's already constrained by previous constraints (even if they're just variable names) it will tell you you can't do that, but you can, instead, calculate it in terms of the existing constraints. Thus, in the first diagram above, when the sides of a triangle are constrained to be a, b, c it assigns the angle then next free variable name z_0 and computes it as $z_0 = \arccos\left(\dfrac{a^2 - b^2 + c^2}{2ac}\right)$.

In the second diagram I depicted the ambiguous ASS construction where $\overline{FG}$ was constrained to have the length as $\overline{EF}$, producing two triangles with θ opposite d. The shorter length $\overline{DG}$ can then be computed by first trying to constrain it. You can see how to compute the value z_1 looking at the right triangle formed by a perpendicular from F to $\overline{DE}$.

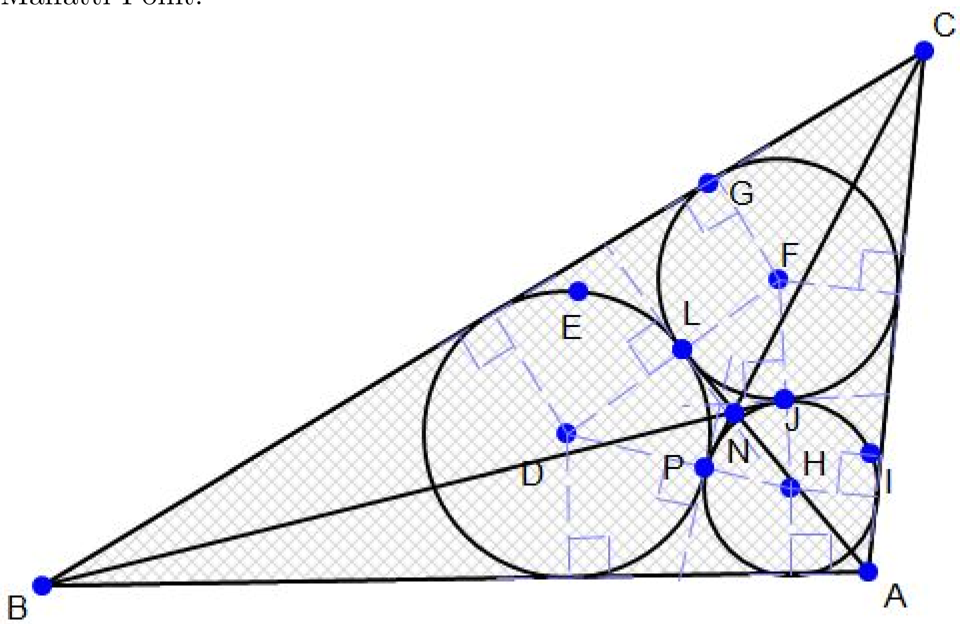
The challenge of constructing a regular hexagon by imposing constraints on an arbitrary hexagon was a lot of fun! Here's the diagram I ended up with:



I'm not sure why the z_0 calculation cropped up at the very end, when I successfully imposed the fourth congruent side on the regular hexagon in the upper left.

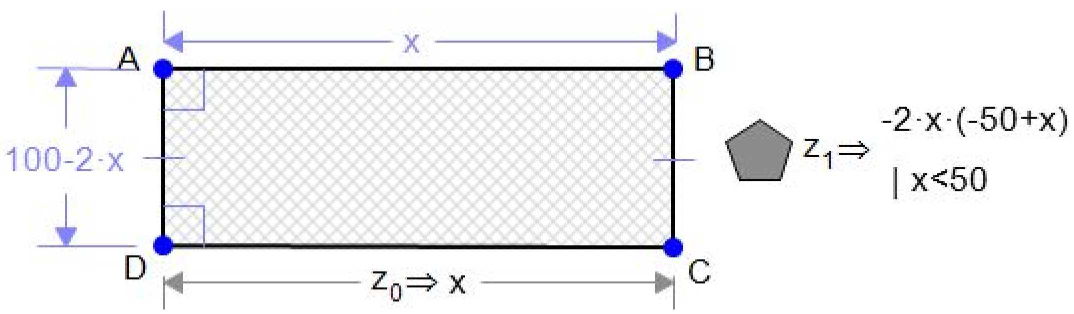
A simpler approach (more akin to Euclid's) would have you inscribe the hexagon in a circle and constrain the sides to have the same length as the radius of the circle.

Using the parallel constraints it was easy to construct a triangle containing three mutually tangent non-overlapping circles and find the point of concurrency where segments from each vertex to the point of tangency of the two remote circles intersect. This is the "First Ajima-Malfatti Point."



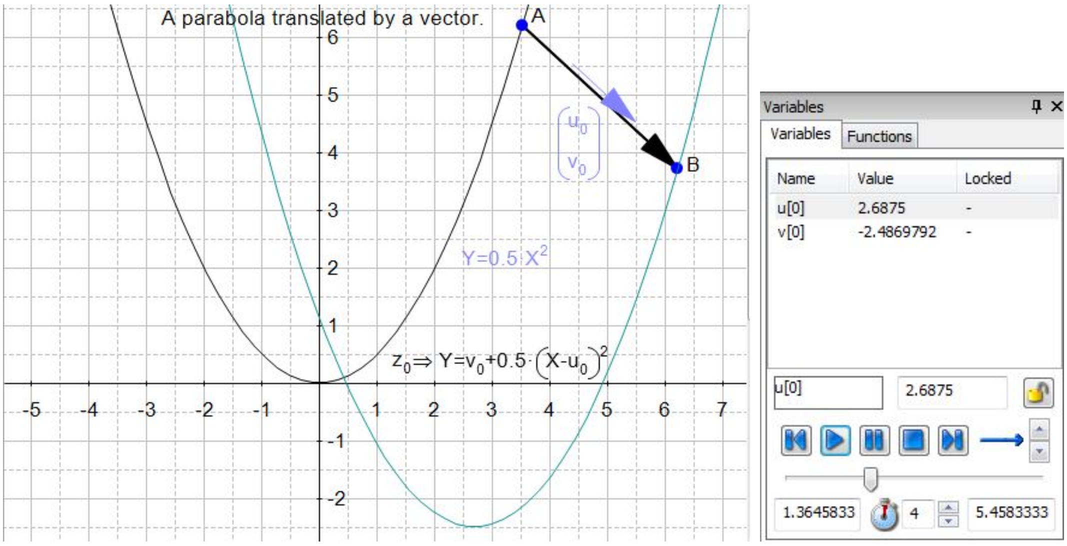
Here's a classic intermediate algebra problem that can be visualized in GE:

> A farmer wishes to use 100 feet of fencing to enclose a rectangular area. Being clever, he plans to use the fence for three sides of the area, and an existing wall of sufficient length for the fourth wall. What is the maximum value of the area contained?



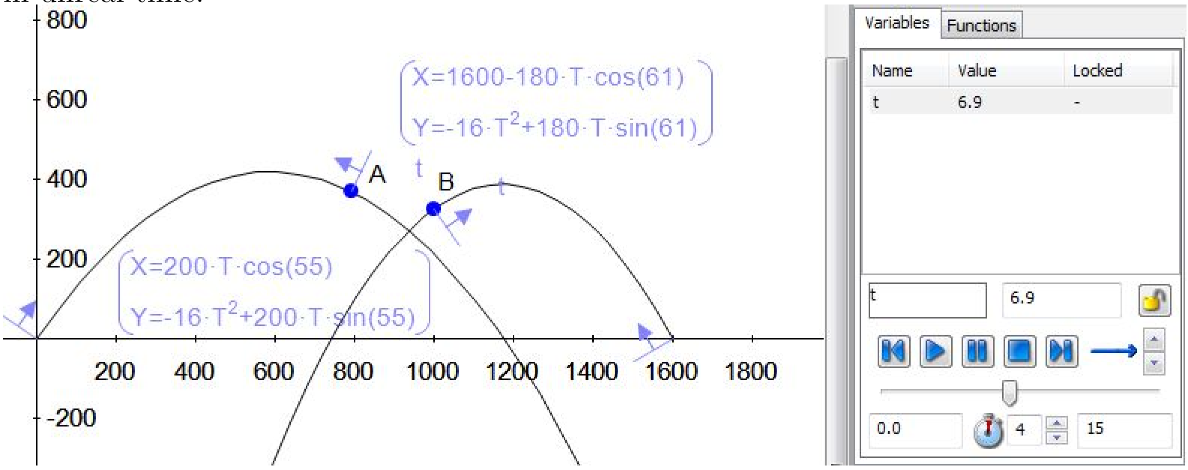
Construct a rectangle using the polygon tool and then constraining three of the corners to be right angles and a pair of opposite sides congruent. Then constrain the opposite sides of the fencing to be x and the side in between to be $100 - 2 \cdot x$. Then in the variables window limit x to range between 0 and 50, select the rectangle, right click on it and choose "calculate area" from the pop-up window. GX won't, as far as I know, then compute the max of this function, but it's a good start!

Then I used the function tool to plot $y = 0.5 \cdot x^2$, entered a vector $\overrightarrow{AB}$, used the translate tool to shift the parabola according to the vector and then asked for the formula of the translated parabola. Not bad!

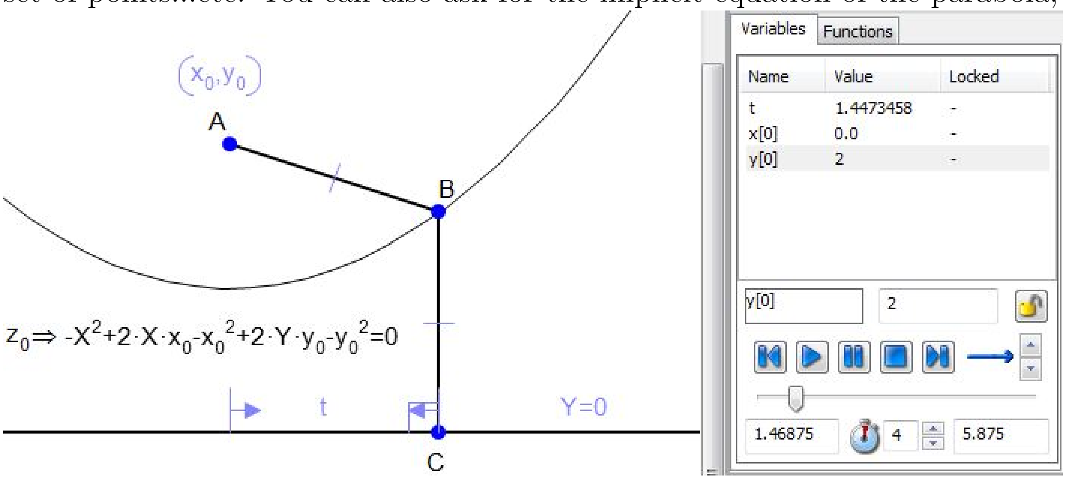


Using the variables dialog box (shown at the right above) you can define intervals for various variables and animate them in various ways. Very natural and illuminating, Dr. Otter!

Or you can set up a couple of projectiles heading towards an uncertain collision using the "point proportional along curve" constraint and the slider to visualize the bombs not colliding in unreal time!



Here's another construct using the geometric definition of a parabola with the directrix constrained to $y = 0$, the focus at (x_0, y_0), $\overline{AB} \cong \overline{BC}$, $\overline{BC} \perp$ the directrix and the locus of points for B is constructed. You can animate the construction to show how the parabola is truly the set of points...etc. You can also ask for the implicit equation of the parabola, as I have done:



Undecidability in Number Theory

The title hyperlink is to the paper Poonen wrote which is pretty close to the sequence of three talks he delivered at MathFest concerning Hilbert's Tenth Problem and larger issues of undecidability. The paper is quite readable so I won't try to summarize it here...well, ok, a little. He defines *diophantine* sets and gives the example that $\mathbb{N} \in \mathbb{Z}$ is diophantine since $a \in \mathbb{Z} \Leftrightarrow (\exists x_1, \ldots x_4 \in \mathbb{Z})\, x_1^2 + \cdots + x_4^2 = a$, meaning, I gather, that every natural number can be written as the sum of four squares:$1 = 1^2, 2 = 1^2 + 1^2, 3 = 1^2 + 1^2 + 1^2, 4 = 2^2, \ldots.$

A subset of $\mathbb{Z}$ such as the integers that are expressible as a sum of three cubes is *listable* if there is an algorithm that will find them. All diophantine sets are listable.

A subset of $\mathbb{Z}$ is *computable* if there is an algorithm for deciding whether a given number is in the set or not. Every computable number is listable, but not so the reverse.

The *halting problem* asks for an algorithm that takes as input a computer program p and an integer x, and outputs YES or NO, according to whether program p run on input x eventually halts (instead of entering an infinite loop, say). Turing proved that the halting problem is undecidable by contradiction: Assume there is a program Halt(P I) that solves the halting problem, Halt(P,I) returns True if and only P halts on I. Then given this program for the Halting Problem, we could construct the following string/code Z:

```
Program (String x)

If Halt(x, x) then
     Loop Forever
Else Halt.

End.
```

Consider what happens when the program Z is run with input Z

Case 1: Program Z halts on input Z. Hence, by the correctness of the Halt program, Halt returns true on input Z, Z. Hence, program Z loops forever on input Z. Contradiction.

Case 1: Program Z loops forever on input Z. Hence, by the correctness of the Halt program, Halt returns false on input Z, Z. Hence, program Z halts on input Z. Contradiction.

As a corollary, there exists a listable set that is not computable.

Proof. Let A be the set of numbers 2^p3^x such that program p halts on input x. By Turing's theorem, A cannot be computable. On the other hand, here is a program that prints A: loop over $N = 1, 2, \ldots$ and during iteration N, for each $p, x \leq N$, run program p on input x for N steps, and print 2^p3^x if the program halts within these N steps.

The DPRM theorem (Davis, Putnam, Robinson, Matiyasevich 1970). A subset of $\mathbb{Z}$ is listable if and only if it is diophantine.

This then provides a negative answer to H10 (Hilbert's tenth problem). I would explain in more detail, but then you would have to award me with a PhD, and nobody wants that. Poonen mentioned how Markov in 1958 proved that the problem of deciding whether two finite simplicial complexes are homeomorphic is undecidable, which was amusing, not for the fact of that, but because this Markov proved to be part of a chain of famous mathematician Markovs...

SIMIODE

SIMIODE is a Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations. The minicourse in SIMIODE was led by Jessica Libertini, Virginia Military Institute, and Brian Winkel, US Military Academy. Well, mostly Winkel, who reminded me how lavish the West Point salaries are and how tiny their work load is...but I digress.

The idea is to create a repository of teaching/research materials related to modeling with ODEs. I worked with John Thoo of Yuba College on a few projects. We started with this activity:

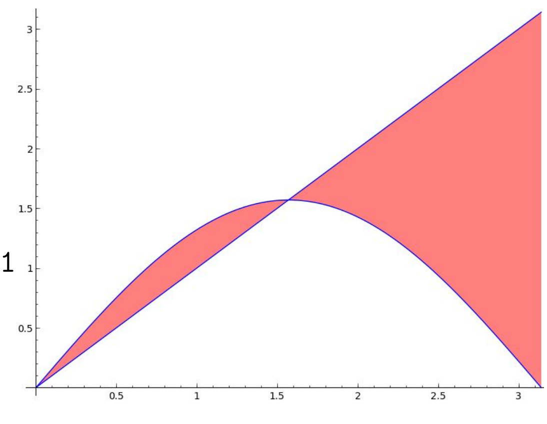
> GOAL: Represent the function $f(x) = x$ on the interval $[-\pi, \pi]$ as a sum of trigonometric functions of the form $a_n \cdot \sin(nx), n = 1, 2, 3, \ldots$

If n wasn't constrained to the integers, then you could do something like $y = 9999 \cdot \sin(x/9999)$, but that's cheating.
The question is how to measure? Suppose we want to minimize the area between the two curves:

$$A = \int_0^{\pi} |a_1 \sin(x) - x| dx$$

a	1.57	
deltaX	0.15707963267948966	
integral	2.7142662970200084	
x	a*sin(x)	abs(x-a*sin(x))
0	0	0
0.15707963267948966	0.24560211011316246	8.8522477433672808E-2
0.31415926535897931	0.48515668116866745	0.17099741580968814
0.47123889803846897	0.71276508459108845	0.24152618655261948
0.62831853071795862	0.92282284609918286	0.29450431538122424
...	...	...
2.6703537555513241	0.71276508459108856	1.9575886709602357
2.8274333882308138	0.48515668116866761	2.3422767070621462
2.9845130209103035	0.24560211011316266	2.738910910797141
3.1415926535897931	1.9234830742065336E-16	3.1415926535897931
	sum of f values:	17.279555921538758

The spreadsheet partially shown above tabulates $x, a\sin(x), |x - a\sin(x)|$ values with $0 \leq x \leq \pi$ and $\delta x = \frac{\pi}{20}$. By entering various values for a in cell B2 we arrive at $a = 1.57$ (pi/2?) as an approximate value that minimizes the error integral. Using the command `plot([x,1.57*sin(x)],(x,0,pi),fill={0:[1` in Sage we get the plot:



As with a sum of square error, it may work better to measure the error with

$$E(a) = \int_0^\pi (a\sin(x) - x)^2\, dx$$

In that case, $a = 2$ minimizes the error.
One advantage of using the square instead of absolute value is that the symbols work out clean. As usual, you need to define your variables: `a=var('a')` and `x=var('x')` and the integrand function, `f=(x-a*sin(x))∧2`. Then the integral is elementary and we have
`f.integral(x,0,pi)=1/3*pi∧3+1/2*pi*a∧2-2*pi*a` whence
`derivative(f.integral(x,0,pi),a)` produces `-2*pi+pi*a`.

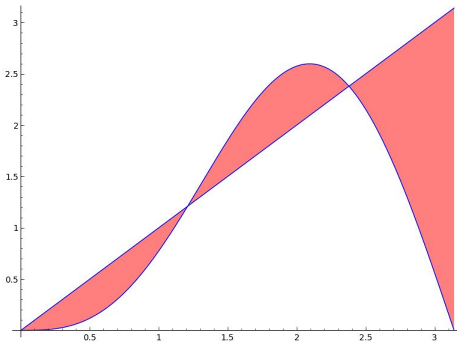
It's also easier to work the symbols for the square error if you add a higher frequency term and try to approximate

$$x \approx a_1\sin(x) + a_2\sin(2x)$$

with the error

$$E(a_1,a_2) = \int_0^\pi (x - a_1\sin(x) - a_2\sin(2x))^2\, dx = \frac{\pi^3}{3} + \frac{\pi a_1^2}{2} + \frac{\pi a_2^2}{2} - 2\pi\cdot a_1 + \pi\cdot a_2$$

Setting the partials to zero yields $a_1 = 2, a_2 = -1$ and the area of this error is shown at right. Taking this further with Sage, enter
`a1,a2,...,a5=var('a1','a2',...,'a5')`
`g=(x-a1*sin(x)-...-a5*sin(5*x))^2`
`g.expand()` to see the various types of terms involved in computing the integral symbolically.



If you look at the expansion as the dot product of the vector $\vec{a} = \langle a_1, a_2, \ldots, a_n\rangle$ with $\vec{v} = \langle \sin(x), \sin(2x), \ldots, \sin(nx)\rangle$ and expand

$$E(\vec{a}) = \int_0^\pi (x - \vec{a}\cdot\vec{v})^2\, dx$$

you have four different types of integrals:

- The constant $\int_0^\pi x^2 dx = \dfrac{\pi^3}{3}$
- Multiples of a_k like $2a_k \int_0^\pi x\sin(kx)dx = 2a_k(-(-1)^k\pi)/k$
- Multiples of a_ia_j where $i \neq j$ like $2a_ia_j \int_0^\pi \sin(ix)\sin(jx)dx$
  $= 2\dfrac{a_ia_j}{2}\int_0^\pi \cos((i+j)x) - \cos((i-j)x)dx = \dfrac{a_ia_j}{2}\left(\dfrac{\sin(i-j)}{i-j} - \dfrac{\sin(i+j)}{i+j}\right)\Big|_0^\pi = 0$

- Multiples of a_i^2 like $a_i^2 \int\limits_0^\pi \sin^2(ix)dx = a_i^2\pi/2$

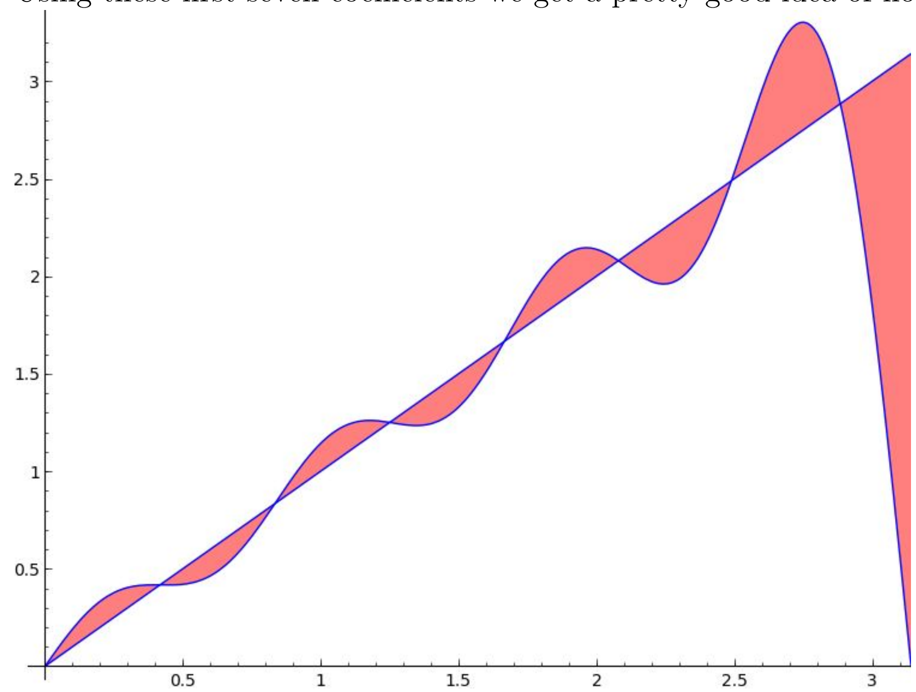
Putting it all back together, we'd like to minimize

$$E(\vec{a}) = \frac{\pi^3}{3} + 2\sum_{i=0}^{n} a_i \int\limits_0^\pi x\sin(ix)dx + a_i^2\pi/2$$

so we set the partial derivatives to zero and get

$$\pi a_i - 2\frac{(-1)^i\pi}{i} = 0 \Leftrightarrow a_i = \frac{2(-1)^i}{i}$$

Using these first seven coefficients we get a pretty good idea of how this is going:



Here's another goodie we worked on.

M&M - Death and Immigration

> STATEMENT Consider a regular size bag of M&M candies, not peanut, just regular. Usually there are about 55 pieces in each bag. They are of different colors, but each piece has an "m" pressed on one side and not the other. Hence, there are distinctions between the sides. Let us conduct an experiment (a life and death experiment) on the M&Ms.

Gently shake them out onto the desk (you might want to use a paper plate to catch the M&Ms and keep them clean as well) to determine for each M&M if it lives or dies. If the m shows on top this M&M dies, otherwise there is life for this M&M. Upon death you should remove the M&M from the population (set these aside as we will need them for another experiment), count and note down the number of M&Ms who survive in Table, and thus put fewer M&Ms back into your container for the next iteration. Do this over and over and record your results.

1. Make a count of the M&Ms on your plate.

2. Set aside the M&Ms with an m facing up (be careful with the yellow ones!)
3. Put the M&Ms remaining in a cup, counting them.
4. Add 10 more from the rejection pile and write down the total.
5. Gently shake these out on to the paper plate and go to step 2.

Modeling M&Ms Population	
Iteration	# M&Ms at start of iteration
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Table 1: Modeling death of M&Ms.

a) State your assumptions about the physical activity.
b) Offer up a description of what should happen. Care to make a prediction?
c) In Table 1 record what happened and compare with what you thought would happen.
d) Compare your description/prediction with what actually happened.
e) Indicate which of your assumptions were reasonable and played a role in the experiment.

The investigation will steer students towards the difference model

$$b_{n+1} = b_n/2 + 10$$

whose behavior is easily modeled with a spreadsheet.
This got me and John Thoo thinking. What if the new survivors are effected by two previous generations lie so: $b_{n+2} = b_{n+1}/2 + b_n/2$. This can be modeled by

$$\begin{pmatrix} b_{n+2} \\ b_{n+1} \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} b_{n+1} \\ b_n \end{pmatrix}$$

If you diagonalize the matrix like so

$$\begin{pmatrix} 1/2 & 1/2 \\ 1 & 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1/2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$$

You can see that the $\lim_{n\to\infty} = \frac{2}{3}b_0$

Any road, SIMIODE is a fine repository of ODE models and I expect to use it in the spring when scheduled for Math 2C. Oh, and Dr. Winkel recommended Martin Braun's *Differential Equations and their Applications* which I managed to get a copy of over at Powell's (score!). It's chock full of great applications.

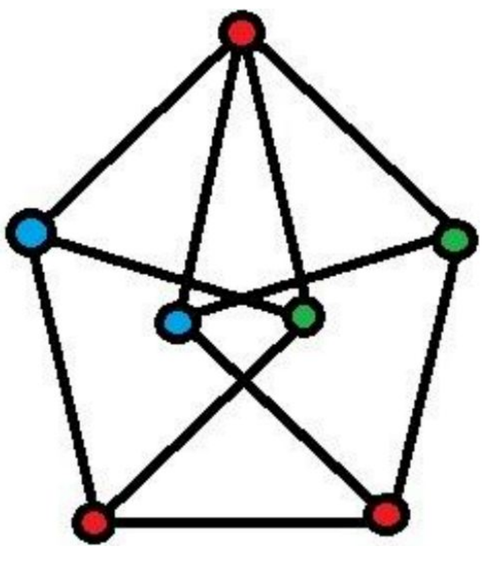
Many of the talks I attended involved geometry of some sort. For example, Sara Billey University of Washington (billey@math.washington.edu)

What is the Value of a Computer Proof in Research and Teaching?

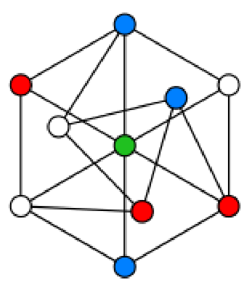
Dr. Billey introduced some of the history of computer assisted proofs, modern applications, and how you can incorporate this technique into your every day life. Beyond the famous 4-Color Theorem, computer assisted proofs are found in hypergeometric series, geometry of Kepler's conjecture, and algebraic geometry related to Schubert varieties. She speculates that computer assisted proofs will be taught right along side the techniques of induction and proof by contradiction.

Mike Krebs, of Cal State, LA gave the talk,

Coloring the Plane with Rainbow Squares

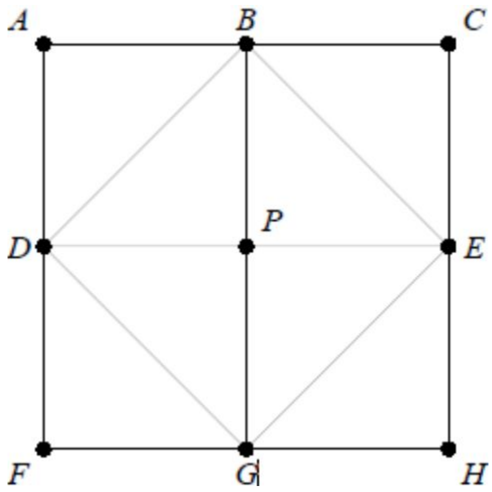


The Hadwiger-Nelson problem:
Assign every point in the plane a color so that no two points of distance 1 from each other have the same color. What is the smallest number of colors needed?
The question can be phrased in graph theoretic terms as follows. Let G be the unit distance graph of the plane: an infinite graph with all points of the plane as vertices and with an edge between two vertices if and only if there is unit distance between the two points. Then the Hadwiger–Nelson problem is to find the chromatic number of G.
The first thing to notice is, at least three colors are needed.



The fact that at least four colors are needed is demonstrated by both the Moser Spindle and Solomon W. Golomb's ten vertex graph (at right.)
A variation on the Hadwiger-Nelson problem assigns every point in the plane a color

and calls a square a rainbow square if its vertices all have different colors. What is the smallest number of colors needed so that every unit square is a rainbow square?



...and while we're posing, how about a Putnam problem:
A function $f : \mathbb{R}^2 \to \mathbb{R}$ has the property that $f(A) + f(B) + f(C) + f(D) = 0$ whenever $A, B, C,$ and D are the vertices of a square. Must f be the zero function?
Answer: Yes.

Proof: $f(A) + f(B) + f(P) + f(D) = 0$
$f(B) + f(C) + f(E) + f(P) = 0$
$f(P) + f(E) + f(H) + f(G) = 0$
$f(D) + f(P) + f(G) + f(F) = 0$
$f(A) + f(C) + f(H) + f(F) + 2f(B) + 2f(E) + 2f(G) + 2f(D) + 4f(P) = 0 \Rightarrow f(P) = 0$

Triangles with Trisectible Angles

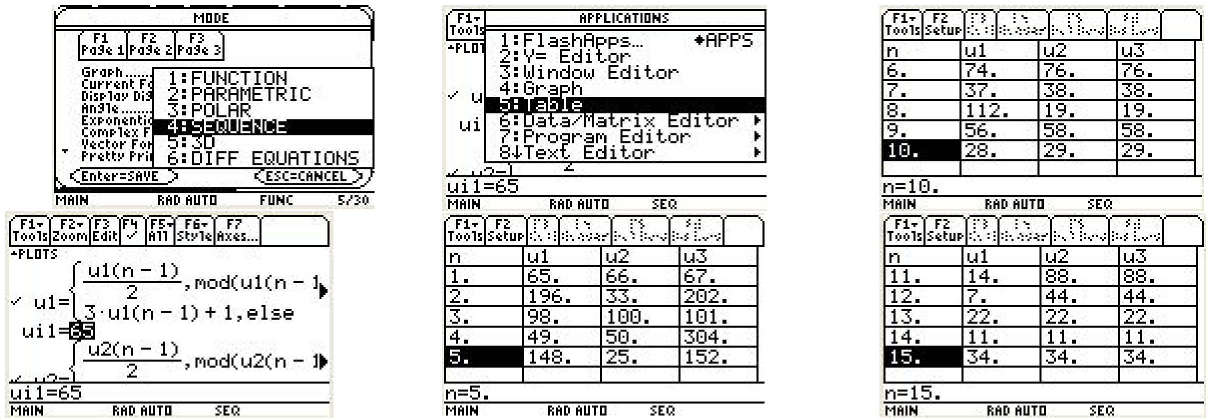
Russell Gordon of Whitman College in Walla Walla, WA delivered his paper from the June/14 Mathematics Magazine wherein he proves the following are equivalent:

- the angle $\theta/3$ is constructible
- $\cos(\theta/3)$ is a constructible number
- the polynomial $4x^3 - 3x - \cos(\theta)$ has a rational root
- there is $t \in (-1, 1)$ such that $\cos(\theta) = 4t^3 - 3t$

Which all seems obvious enough - in fact, the whole paper is approachable from a precalc perspective. He defines *residual* r thusly: Let $\theta \in (0, pi)$ be such that $\cos(\theta) = m/n$ where $m, n \in \mathbb{Z}$ and express $n^2 - m^2 = j^2 r$ where $j, r \in \mathbb{Z}$ and j is square free.

Exploring Five Integer Sequences Related to the Collatz Problem

Jay Lawrence Schiffman showed how to use the TI Voyager 200 and Mathematica to explore the Collatz conjecture. On the TI200 you enter sequences:
```
u1(n)=when(mod(u1(n-1),2)=0,u1(n-1)/2,3*u1(n-1)+1
ui1=65)
u2(n)=when(mod(u2(n-1),2)=0,u2(n-1)/2,3*u2(n-1)+1
ui2=66)
u3(n)=when(mod(u3(n-1),2)=0,u2(n-1)/2,3*u3(n-1)+1
ui3=67)
```
Here are some screen shots (actually the TI89) showing this:



Alternatively, define on the home screen `collatz(n)=when(mod(n,2)=0,n/2,3*n+1`

There were many other great talks I saw, but I need to start writing syllabi for the fall14! So here are the abstracts of the geometry talks I liked:

Deirdre Longacher Smeltzer Eastern Mennonite University (deirdre.smeltzer@emu.edu)

A Proof of Ptolemy's Theorem via Inversions

Deirdre Longacher Smeltzer Eastern Mennonite University Ptolemy's theorem, attributed to second century Greek mathematician Claudius Ptolemaeus, gives necessary and sufficient conditions for one to be able to inscribe a given quadrilateral in a circle. A standard proof involves using inscribed angles and similar triangles. A more elegant and modern proof utilizes an inversion in the plane and resulting properties to establish a generalization of the theorem.

Archimedes' Twin Circles in an Arbelos

Dan C. Kemp South Dakota State University (dan.kemp@sdstate.edu)
Proposition 5 from Archimedes' Book of Lemmas was popularized in 1954 by Leon Bankoff as a surprise in an arbelos. An arbelos consists of three mutually tangent semicircles with diameters on a common line and lying on the same side of that line. Archimedes asserts that in an arbelos, the two circles that are tangent to two of the semicircles and the common tangent of the smaller semicircles are congruent. Archimedes' synthetic proof, suitable for presentation in a geometry course, is given. Archimedes' proof is historically interesting because it contains the first known reference ('...by the properties of triangles...') of the altitudes of a triangle being concurrent. Also a modern proof using analytic geometry will be presented. If time permits, further discussion of the twin circles of Archimedes will be given.

Finally, I saw Persi Diaconis deliver the Martin Gardner Centennial Lecture. What can I say? It was very moving. You had to be there