Alice in Numberland

The first presentation, "Alice in Numberland," by Alice Silverman of UC Irinve, was well attended, fillint

1 Thursday

I started Thursday (after discovering there was no good source of coffee at the Duke Energy Convention Center) in the "Plug and Play Data Science Lessons" which is described as

> ...papers includ[ing] data science lessons that attendees can seamlessly incorporate into courses such as Finite Math, Calculus, Linear Algebra, Discrete Mathematics, Mathematical Modeling, and others. Presentations include such elements as an overview of the lesson, student learning objectives, assessments, and a summary of the effectiveness of the lesson if available.

The first talk was by Tia Sondjaja of NYU. What she said about using Python, Cocalc and Jupyter resonated with me, since I've been investigating the possibilities of those open platforms for years now. She talked about classifying text files by genre, especially files involving movie scripts, but with much more general applications. She puts her syllabus on Github.

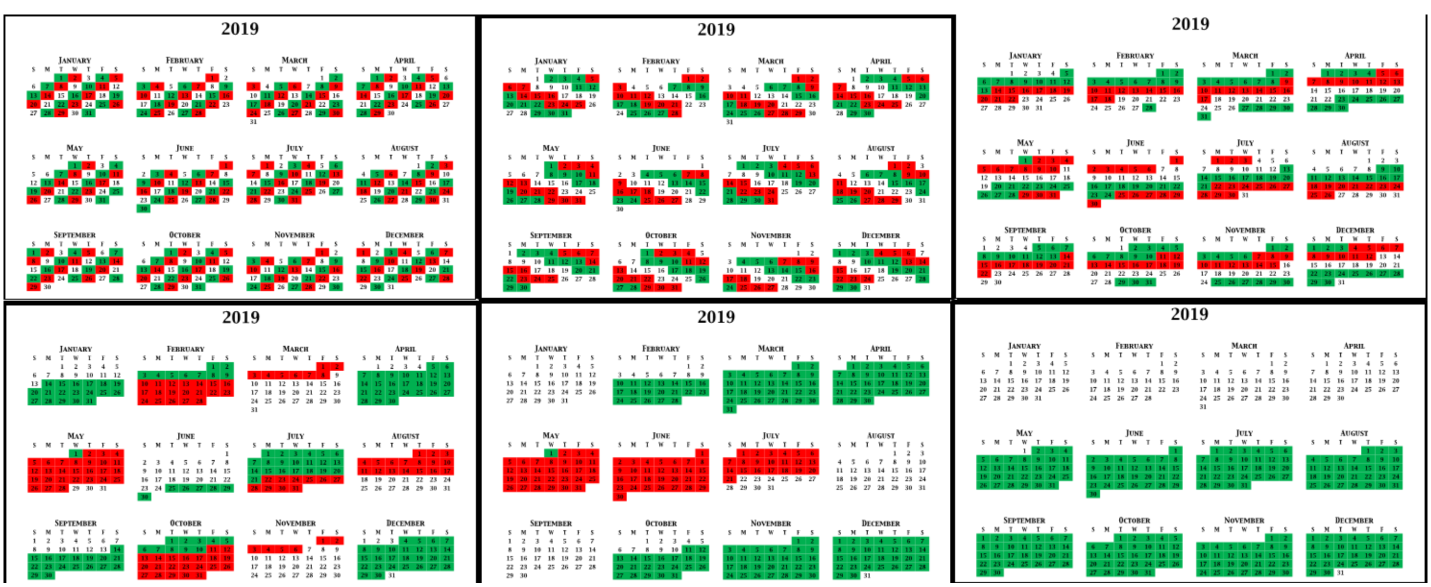
It calls to mind what UC Berkeley is doing with introductory math courses for all students: Data 8: Foundations of Data Science, which strikes me as a better model for the "statways" path than Sociology 3...

After the first session in "Plug and Play," I'd had enough, so I did not resist the irresistable pull to the " Recreational Mathematics: Puzzles, Card Tricks, Games, Gambling and Sports" thread.

Guess My Birthday - An Original Mathematical Magic Trick

This talk by Jon Stadler of Capital University in Columbus, OH. The procedure is thus:

1. Pick any date, both the month and day, from 2019. Choose a birthday or other special day if you need inspiration. This will be your secret day (shh!). Note that 2019 is not a leap year.
2. There are six different calendars provided. Each calendar represents a different possible rotation of exercises. Days in which I will ride a bike are denoted in red and days that I will run are denoted in green. Uncolored days represent days of rest (ahhh!).
3. For each calendar, if your secret day is listed in red, clip the  to that calendar. If the secret day is listed in green, clip the  to the calendar.
4. When you have checked all of the calendars, stack them and get my attention to get them from you.



Note that each calendar gourps the days in groups of $3^k, k = 0, 1, 2, 3, 4, 5$. This trick involves the balanced ternary system where for every integer n, there are unique integers $a_0, a_1, \ldots, a_{k-1}, a_k \in \{-1, 0, 1\}$ $n = a_k 3^k + a_{k-1}3^{k-1} + \cdots a_1 3 + a_0$.
Example.
$2019 = 3^7 + (-1)3^5 + 3^4 + (-1)3^2 + 3^1 = +0 - +0 - +0 = u0nu0nu0$
The largest integer that can be expressed in the balanced ternary system using powers of 3 up to 3^k is
$1+3+3^2+\cdots+3^k = \dfrac{3^{k+1}-1}{3-1} = \frac{1}{2}(3^{k+1}-1)$, so we get almost every day of the year with $k = 5$: $1+3+9+\cdots+243 = 364$.
By using December 31 = 000000, we can represent each date of the year with the day of the year on which it occurs.
The first days of each month will serve as benchmarks:

Jan 1 = 1= = $+00000 = u00000$,

Feb 1 = $32 = -1 - 3 + 9 + 27 =$ $= - - + + 00 = nnuu00$,

Mar 1 = $60 = -3 + 9 - 27 + 81 =$ $= 0 - + - + = 0nunu0$,

Apr 1 = $91 = 1 + 9 + 81 =$ $= +0 + 0 + 0 = u0u0u0$,

May 1 = $121 = 1 + 3 + 9 + 27 + 81 =$ $= + + + + +0 = uuuuu0$,

June 1 = $152 = 3 + 27 - 81 + 243 =$ $= 0 + 0 + -+ = 0u0unu$,

July 1 = $182 = -1 + 3 - 9 + 27 - 81 + 243 =$ $= - + - + -+ = nununu$,

Aug 1 = $213 = -3 - 27 + 243 =$ $= 0 - 0 - 0+ = 0n0n0u$,

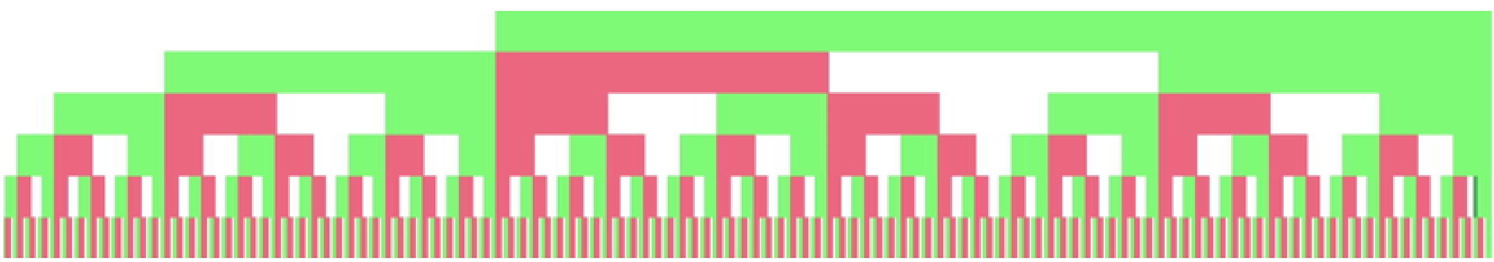
Sep 1 = $244 = 1 + 243 =$ $= +0000+ = u0000u$

Oct 1 = $274 = 1 + 3 + 27 + 243 =$ $= + + 0 + 0+ = uu0u0u$

Nov 1 = $305 = -1 + 9 - 27 + 81 + 243 =$ $= -0 + - + + = n0unuu$

and Dec 1 = $335 = -1 + 3 + 9 + 81 + 243 =$ $= - + +0 + + = nuu0uu$.

And here's a fractalish interpretation of the balanced ternary system:

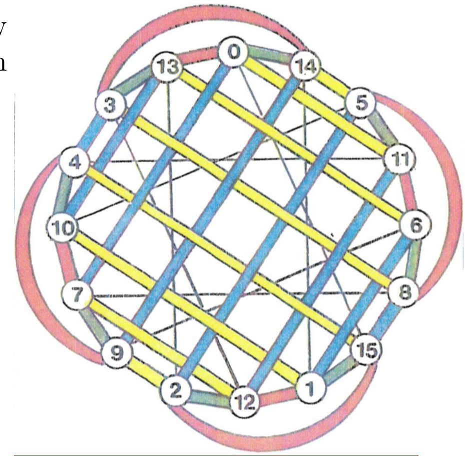


Example: If the subject has a birthday of August 15 then they will consult the 6 calendars to produce $-1+3+9-27+243 = 227$ So these items will be placed on the calendars allowing the magician to compute 227 and so the month is $1 + \lfloor \dfrac{227}{30} \rfloor = 8$ and the day is $227 \mod (30) - 2 = 15$ where, and this "-2" is due to the quirkiness of the months. The pattern goes, Jan "+0"', Feb '−1", Mar "+1", Apr "+0", May "+0", Jun "−1", Jul "−1", Aug "−2", Sep "−3", Oct "−3", Nov "−4", Dec "−4". You've got to add up signed powers of 3 in your head and remember those quirky offsets. Another difficulty with the trick is that birthdays Dec 31–Jan 2 are going to seem not to remarkable a feat.

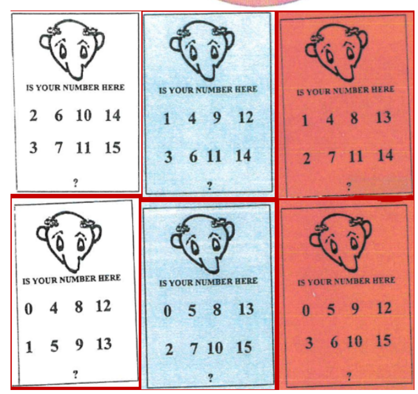
Five Card Study: A Magic Divination

Jeremiah Paul Ferrall is a fixture in recreational math for over five decades now, with a treasure of puzzles/games attributed to him. Here he is at the top of his game, even though he needs a walker to get there...

The effect: The magician shows the subject the road-map wheel with the 16 nodes, or stations, and the five colored routes between them. He explains its use with an example. "Suppose we decide to travel the red, green and yellow lines and choose to start at Station 3. We could go red to Station 14, green to Station 0 and, finally, end at Station 11 by traveling yellow.



After the subject is familiar with the wheel, five colored cards with numbers on them are displayed, while the magician explains, "All 16 numbers are on one side or the other of each of these cards," adding, "You may turn the five cards so that any combination of numbers is showing."

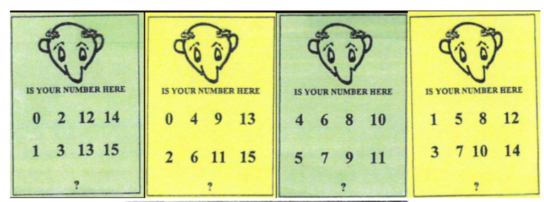


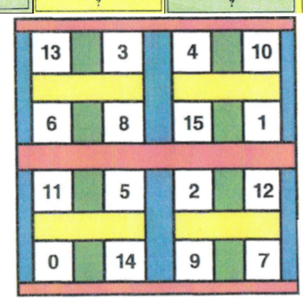
When the subject is satisfied with the placement of the five cards, she is asked to secretly jot down one of the 16 stations (0–15). The magician had previously written down a prediction for the station number.

Privately, the subject notes on which of the colored cards their number is written, and, using these colors as routes (traveling the black line if the color on which his number appears is white) a path is traversed from the starting station to another station, at which point the player announces, "I have arrived."

Even though the magician does not know the subject's start or the path followed or where it ends, the magician then directs the subject to follow a path beyond the current position. This journey is bound to end up at the magicians predicted destination.

Another effect: The magician constructs the 4x4 magic square with the numbers $0, 1, 2, \dots, 15$ and marvels at the many ways the magic sum 30 appears on the board.





"Mathematicians call such a fecund magic square 'Most-Perfect' and the ancients considered such squares to be endowed with mystical powers," he says. The magician claims to have studied the powers of the square and proceeds to demonstrate.

"Chose one of the 16 numbers–do not tell me which one it is of course. I now show you fice colored cards with the numbers of the square printed on one side or the other of each of them."

He holds each card in turn up to the subject's face, deliberately showing both sides, and places the cards down in front of the subject.

The magician allows the subject to turn over any of the five cards he cares to and adds, "I am going to ask you five simple yes-no questions and to make it harder on me, I want you to secretly choose to be either "convivial" and always tell the truth, or, to be "contrary" and always lie. That is, to tell five straight truths or five straight lies to the questions."

The question are all of the form "Is you number here?" for each of the five cards and the "yes" responses are put to one side.

The magician glances at the magic square gird and quickly and correctly names the subject's number. (You may wish to have the subject write his number down earlier for verification.)

The method: No matter how the five colored cards are turned there will always be exactly one number of the 126 that either appears on all five cards or fails to appear on all five cards. This is called the "forced" number. Let us suppose as an example that the magician chooses 2 to be the forced number.

Turn the cards so that 2 appears on each one. Suppose the subject selects the number 6, and decides to lie. He will say his number appears on the red, green and blue cards. On the magic square, the magician mentally starts at 2 (the force), crosses the red edge to 15, the blue edge to 8, and the green edge to 6 the chosen number. If at any time the edges of the square are reached, the magician jumps to the other side of the square for blue or red yeses. For example, for the chose 6, the magician could have started at the forced 2, got red to 15, then green to 1 and bloue jump acress to 6 as before. If the subject had decided to tell the truth instead of lying, he would have said yes to the yellow and white cards. Starting at 2 as before, the magician crosses yellow to 9 and a yes for white will always mean to make a (unique) diagonal hop–here to 6, (He could have started from 2, diagonally hopped to 13 and then crossedyellow to 6.) The road map wheel works in a similar manner (recalling that a white card yes means travel the black line).

When the magician was holding the five cards up to the subject, he was really identifying his forced number and simply laid the cards down accordingly. If the magician had a chosen 2 as the force, and the subject decided later to turn the blue and yellow cards over, the magician merely changes the force to 14–a blue, yellow move from 2. Once the force number is established the subject when using the wheel will always land on that force no matter whether he lies or not. Hence the magician can always redirect him to any other location the magician wants. And when using the magic square the magivican, who knows the force, can always find the subject's number by crossing the colored edges starting at the force.

Follow-up trick: After performing the above effect the magician scoops up the yes responses (or the noes) and secretly turns them over. This changes the force to the subjects initial choice. In our example, turning yellow and white makes the force 6.

Show the subject the road-map wheel with the 16 nodes and the colored routes between them. Explain its use if this has not already been done. Have the subject choose another number and to note which of the cards his number is on. Ask him to travel from his new number on the colored routes he has selected. He will, much to his amazement, land on his first choice–6 in our example.

The road-map wheel can be used to easily force a specific number on the entire audience by placing the five cards appropriately on an overhead screen.

Either the magic square or the wheel can be regarded as a two-dimensional depiction of a five dimensional hepercube. Neither is a complete graph of the 5-cube since this would be overly confusing in two dimensions. Instead the 32 nodes are reduced by half by regarding each of the 16 numbers as being listed twice on the 32 nodes–once for the convivial and once for contrary. This also reduces by half the number of other parts of the 5-cube, or tesseract, see [F02]. Most-Perfect magic squares are discussed in [OB98] and [P02]. A presentation of the old binary effect is given in [G66].

BIBLIOGRAPHY: F02 Jeremiah Farrell. "Cubist Magic". Puzzlers' Tribute Edited by David Wolfe and Tom Rodgers. A.K. Peters, 2002.
G66 Martin Gardner. New Mathematical Diversions from Scientific American. Simon and Schuster, 1966.
G75 Martin Gardner. Mathematical Carnival, knopf, 1975.
OB98 Kathleen Ollerenshaw and David S. Bree. Most-Perfect Pandiagonal Magic Squares. Their construction and enumeration. The Institute of Mathematics and its APplication. 1998.
P02 Clifford A. Pickover. The Zen of Magic Squares, Circles and Stars. Princeton Univ. Press, 2002.

A Factorial Card Trick

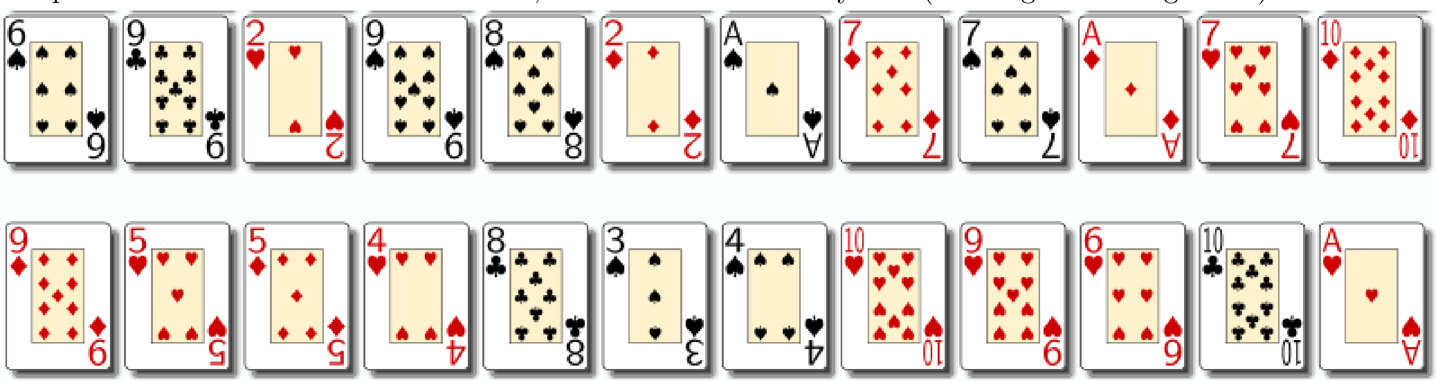
Dr. Thomas Edgar teaches at Pacific Lutheran University near Tacoma, WA, and is the newly elected editor of Math Horizons, where the trick he introduced here will appear in the September, 2019 edition.

Dr. Edgar started by reminding the audience (most of whom have presumably seen it before) of the 21-card trick. We used this during Math Field Day in 2016.

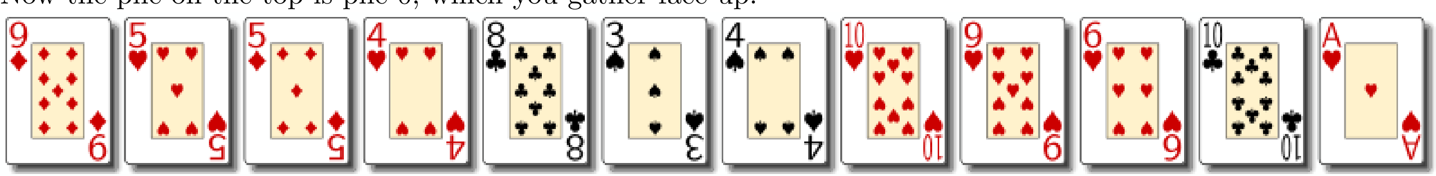
He next described the 27-card trick, which is a clever business like the birthday trick above, making use of the ternary numbers.

This talk introduced a new trick: the 24-card trick. It's similar to the 27 card trick and involves the subject choosing (1) an unspecified card from among 24 cards and (2) specifies a number between 1 and 24.

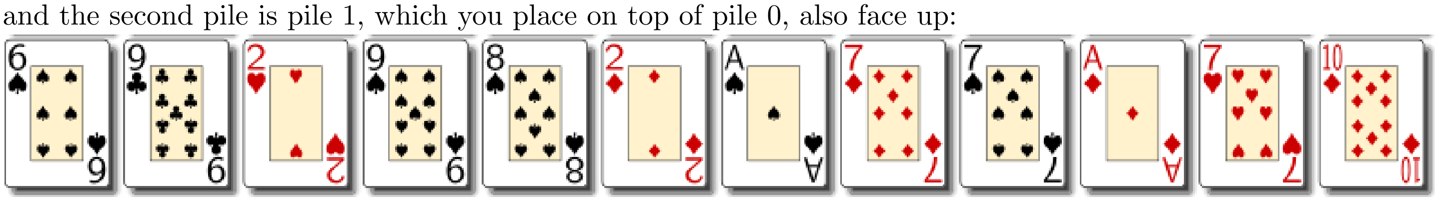
Suppose the subject chooses the ace of spades and the number 18 when presented with these cards. The magician then computes $17 = 2 \cdot 3! \cdot +2 \cdot 2! + 1 \cdot 1! = 122$, in this weird number system (least significant digit first).



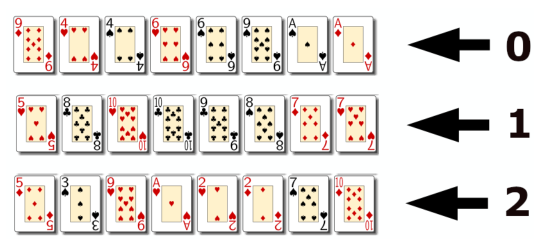
Now the pile on the top is pile 0, which you gather face up:

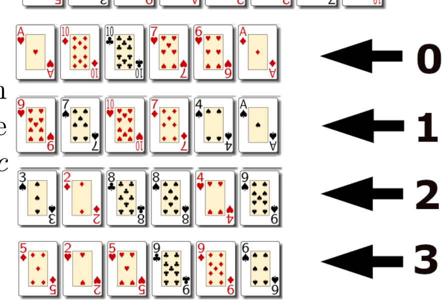


and the second pile is pile 1, which you place on top of pile 0, also face up:



Next, deal the 24 cards out into 3 groups of 8, so the first 12 cards are on the left. Now we know our card is in the 12 cards on the right, and upon asking the subject to point to which group it's in, we find it's in row 0. So we've narrowed the possibilities down to the 4 cards on the upper right. We we gather up the cards again (carefully) and to winnow the possibilities down to 1, deal out the cards in 4 groups of 6 like so:





Note the cards are dealt from the bottom up and left to right. When displayed this way, the ace of spades is in the second row. This is in the *factorial base number system*, sometimes referred to as the *factoradic number system*.