

Putnam Proof Without Words

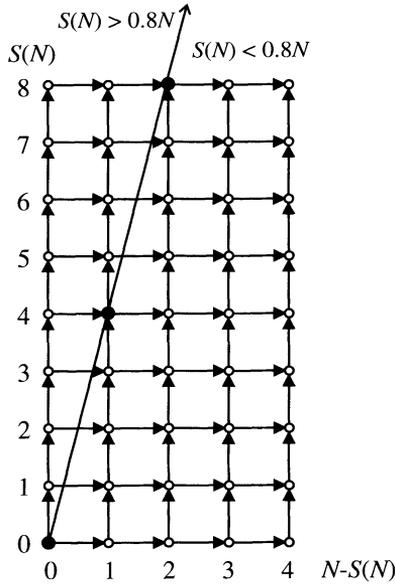
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From the 2004 William Lowell Putnam Mathematical Competition:

Basketball star Shanille O'Keal's team statistician keeps track of the number, $S(N)$, of successful free throws she has made in her first N attempts of the season. Early in the season, $S(N)$ was less than 80% of N , but by the end of the season, $S(N)$ was more than 80% of N . Was there necessarily a moment in between when $S(N)$ was exactly 80% of N ?

Answer: Yes. Proof:



The reader should now also be able to answer the following “riders” that did not form part of the competition question:

1. Answer the same question assuming that Shanille had $S(N) > 0.8N$ early in the season and $S(N) < 0.8N$ at the end.
2. What other values could be substituted for 80% in the original question without changing the answer?