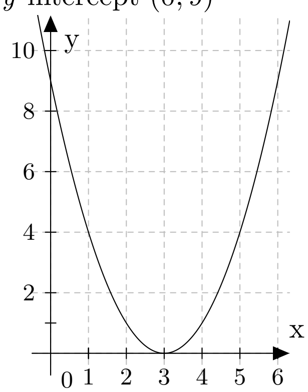
1 Math 40 Exam 4 Solutions

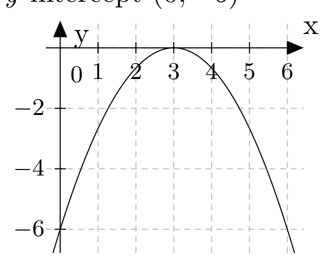
1. Consider the quadratic equation $y = 10x^2 + 21x - 13$
   (a) Specify the values of the coefficients, a, b and c.
   Solution: $a = 10, b = 21, c = -13$
   (b) Compute the value of the discriminant in the quadratic formula.
   Solution: $D = b^2 - 4ac = 21^2 - 4(10)(-13) = 441 + 520 = 961 = 31^2$
   (c) Use the quadratic formula to find the x-intercepts of the parabola.
   Solution: $x = \dfrac{-21 \pm \sqrt{961}}{2(10)} = \dfrac{-21 \pm 31}{20} = \begin{cases} \dfrac{-13}{5} & : \text{if we subtract} \\ \dfrac{1}{2} & : \text{if we add} \end{cases}$
   (d) What is the x-coordinate of the vertex?
   Solution: At least two good approaches to finding the x-coordinate of the vertex. You can take the average of the x-intercepts: $x_v = \dfrac{\frac{-13}{5} + \frac{1}{2}}{2} = \dfrac{-13}{10} + \dfrac{1}{4} = \dfrac{-26}{20} + \dfrac{5}{20} = \dfrac{-21}{20}$ or you can use the formula, $x_v = -\dfrac{b}{2a} = -\dfrac{21}{20}$

2. Graph each parabola. Give the coordinates of the vertex and intercepts in each.

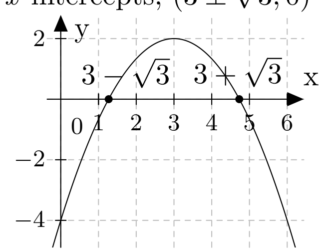
   (a) $y = (x-3)^2$
   Solution:
   Vertex at $(3, 0)$,
   y-intercept $(0, 9)$



   (b) $y = -\dfrac{2}{3}(x-3)^2$
   Solution:
   Vertex at $(3, 0)$,
   y-intercept $(0, -6)$



   (c) $y = 2 - \dfrac{2}{3}(x-3)^2$
   Solution:
   Vertex at $(3, 2)$
   y-intercept, $(0, -4)$
   x-intercepts, $(3 \pm \sqrt{3}, 0)$



3. Find coefficients a, b and c for the parabola $y = ax^2 + bx + c$ that fits the points in the table:

x	-2	1	2
y	13	4	9

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Solution: Plugging the (x, y) pairs into $y = ax^2 + bx + c \Leftrightarrow x^2a + xb + c = y$ we have

$$\begin{aligned} 4a - 2b + c &= 13 \\ a + b + c &= 4 \\ 4a + 2b + c &= 9 \end{aligned}$$

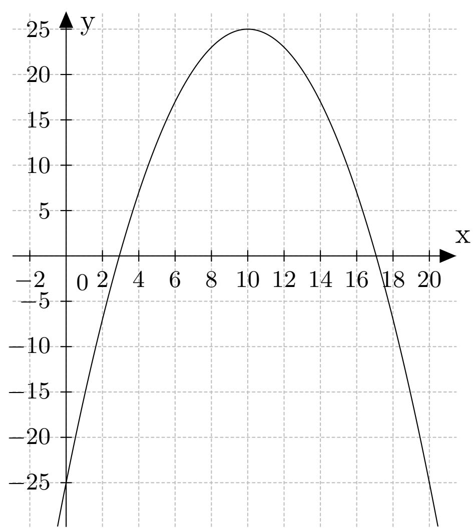
Eliminating c from the first and second, then again from the first and third equations yields

$$\begin{aligned} 3a - 3b &= 9 \\ -4b &= 4 \end{aligned}$$

So $b = -1$ which means that $a = 2$ and so $c = 3$. The equation for the parabola is then $y = 2x^2 - x + 3$ and you can check that it fits the data.

4. A child throws her doll up out a window. The doll starts at a height of 8 meters above the ground and reaches a maximum height of 9 meters when it's 1 meter from the house.
   (a) Write an equation for the height of the doll in terms of its distance from the house.
   Solution: The vertex is at $(1, 9)$ so we can write $h = a(d-1)^2 + 9$. To determine a note that when $d = 0$, $h = 8$ so $8 = a(0-1)^2 + 9 \Leftrightarrow a = -1$. So $h = 9 - (d-1)^2$
   (b) How far from the house will the doll hit the ground?
   Solution: Set $h = 0$ and solve for d: $9 - (d-1)^2 = 0 \Leftrightarrow d - 1 = \pm 3$. Since the doll lands outside the house, we choose $d = 4$ meters.

5. Consider the parabola whose graph is shown at right.



   (a) Find the coordinates of the vertex.
   Solution: The vertex is at $(10, 25)$
   (b) Find the vertex form for the equation of the parabola.
   Solution: $y = a(x-10)^2 + 25$ Since the parabola passes through $(0, -25)$ we can find a by plugging in these coordinates and solving: $-25 = a(0-10)^2 + 25 \Leftrightarrow 100a = -50 \Leftrightarrow a = -\frac{1}{2}$. Thus $y = -\frac{1}{2}(x-10)^2 + 25$
   (c) Find the x-intercepts of the parabola.
   Set $y = 0$ and solve for x: $\frac{1}{2}(x-10)^2 = 25 \Leftrightarrow (x-10)^2 = 50 \Leftrightarrow x = 10 \pm \sqrt{50} = 10 \pm 5\sqrt{2}$

6. Consider the parabola described by $y = -2(x+3)(x-7)$
   (a) What are the x-intercepts of the parabola?
   Solution: The $x$intercepts are at $(-3, 0), (7, 0)$
   (b) What are the coordinates of the vertex?
   Solution: The x-coordinate of the vertex is halfway between the intercepts: $x_v = \dfrac{-3+7}{2} = 2$ and so $y_v = -2(2+3)(2-7) = 50$ Thus the vertex is at $(2, 50)$
   (c) Solve the inequality $-2(x+3)(x-7) \geq 0$. Write the solution in interval notation.
   Solution: The parabola opens downwards from its vertex at $(2, 50)$ and so $y \geq 0$ is x is between the x-intercepts: $-3 \leq x \leq 7 \Leftrightarrow x \in [-3, 7]$

7. Solve each inequality and write the solutions in interval notation.

(a) $(x-1)(x+2) > 0$
Solution:
$x \in (-\infty, -2) \cup (1, \infty)$

(b) $(x-3)^2 - 16 \leq 0$
Solution:
$-4 \leq x - 3 \leq 4$
$\Leftrightarrow x \in [-1, 7]$

(c) $10x^2 + 21x - 13 \leq 0$
Solution:
$x \in \left[\frac{-13}{5}, \frac{1}{2}\right]$