Math 40 - Fall 2013 - Exam 1 (chapter 1) Solutions 9/9/13

1. (16 points) Find the pattern and fill in the table.
Then write an equation for the second variable in terms of the first variable.

x	3	4	9	10	11	12	15
y	$\frac{2}{5}$	1	4	$\frac{23}{5}$			

Solution
We note that the pattern is linear, not surprising since that's what this chapter is all about!
To see this note that the rate of change in y per change in x is constant:

$$\frac{\Delta y}{\Delta x} = \frac{1-\frac{2}{5}}{4-3} = \frac{4-1}{9-4} = \frac{\frac{23}{5}-4}{10-9} = \frac{3}{5}$$

Thus the rate of change in these data is a constant proportion of a change of 3 in y for every change of 5 in x, or, equivalently, a change of $\frac{3}{5}$ in y for every change of 1 in x. To complete the table then we see that a change of 1 in x from 10 to 11 yields a change in y of $\frac{3}{5} = 0.6$ in y from $\frac{23}{5} = 4.6$ to $\frac{26}{5} = 5.2$ and, similarly a change of 1 in x from 11 to 12 yields a change in y of $\frac{3}{5} = 0.6$ in y from $\frac{26}{5} = 5.2$ to $\frac{29}{5} = 5.8$. Finally, $x = 15$ can be seen as a change in x of 5 from $x = 10$, yielding a change in y of 3 from $y = \frac{23}{5} = 4.6$, so the last point in the table has $y = \frac{23}{5} + 3 = \frac{38}{5} = 7.6$.
The table can be filled in with either fractions or their decimal equivalents:

x	3	4	9	10	11	12	15
y	$\frac{2}{5}$	1	4	$\frac{23}{5}$	$\frac{26}{5}$	$\frac{29}{5}$	$\frac{38}{5}$

or

x	3	4	9	10	11	12	15
y	0.4	1	4	4.6	5.2	5.8	7.6

An equation for the line is easy since we know the slope and at least one point we can plug into the slope-intercept formula: $y - 1 = \frac{3}{5}(x-4) \Leftrightarrow y = \frac{3}{5}x - \frac{7}{5}$.

2. Genny is driving from her home in Indio to her parents' home in Blythe, 100 miles away.

Miles Driven	10	30	60	80	90
Miles Remaining					

(a) (8 points) Fill in the table.

Solution:

Miles Driven	10	30	60	80	90
Miles Remaining	90	70	40	20	10

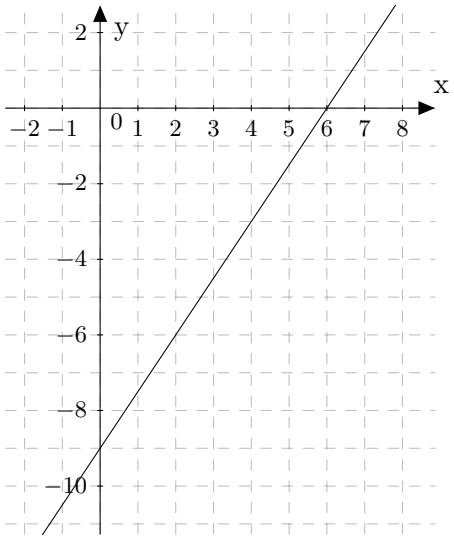
(b) (8 points) Let d stand for the number of miles Genny has driven and r for the number of miles that remain. Write an equation for r in terms of d.
Solution: $r = 100 - d$

3. Consider the equation $4.5x - 3y = 27$.

(a) (8 points) Find the intercepts for the equation and write these as ordered pairs (i.e., in the form (x, y).)
Solution: If $x = 0$ then $-3y = 27 \Leftrightarrow y = -9$, so $(0, -9)$ is the y-intercept.
If $y = 0$ then $4.5x = 27 \Leftrightarrow x = \frac{27}{4.5} = \frac{270}{45} = 6$ so $(6, 0)$ is the x-intercept.

(b) (8 points) Construct a graph for the solution set of the equation showing all points between and including the intercepts.

Solution:

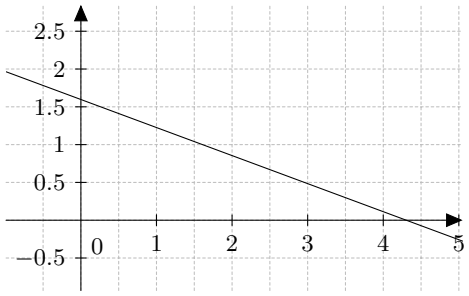


4. Solve for y in terms of x. Simplify your answer either as a fraction in lowest terms or a decimal. Do not approximate.

   (a) (9 points) $-7x + 8y = 36$
   Solution: $-7x + 8y = 36 \Leftrightarrow 8y = 7x + 36 \Leftrightarrow y = \frac{7}{8}x + \frac{9}{2} \Leftrightarrow y = 0.875x + 4.5$

   (b) (9 points) $\frac{2}{3}x - \frac{3}{4}y = \frac{5}{2}$
   Solution: $\frac{2}{3}x - \frac{3}{4}y = \frac{5}{2} \Leftrightarrow -\frac{3}{4}y = -\frac{2}{3}x + \frac{5}{2} \Leftrightarrow y = \frac{8}{9}x - \frac{10}{3} \Leftrightarrow y = 0.\overline{8}x + 3.\overline{3}$

5. A graph for the solution set of $2.3x + 6.2y = 9.9$ is shown below. Use the graph to answer the following questions.



   (a) (4 points) Approximate the value of x where $y = 0$ to the nearest tenth.
   Solution: From the graph it appears that where $y = 0$, $x \approx 4.3$.
   To be sure, $x = \frac{99}{23} \approx 4.3043478260869565217391304347826$ Note the repetend is 3043478260869565217391 (22 digits.)

   (b) (4 points) Approximate the value of y where $x = 0.5$ to the nearest tenth.
   Solution: From the graph it appears that where $x = 0.5$, $y \approx 1.4$. To be sure, $y = \frac{9.9-2.3(0.5)}{6.2} = \frac{8.75}{6.2} = \frac{175}{124} = 1.41\overline{129032258064516} \approx 1.4$

   (c) (4 points) Approximate the value of x where $y = 0.7$ to the nearest tenth.
   Solution: From the graph it appears that where $y = 0.7$, $x \approx 2.5$. To be sure, $x = \frac{9.9-6.2(0.7)}{2.3} = \frac{5.56}{2.3} = \frac{278}{115} = 2.4\overline{173913043478260869565 2}$ ≈ 2.4 is the better approximation.

   (d) (4 points) Give the approximate solution to $x > 4$ to the nearest tenth. **Solution:** From the graph, it appears that $x > 4$ is true (approximately) if $y < 0.1$. To be sure (a gratuitous bit of algebra) If $x = 4$ then $2.3(4) + 6.2y = 9.9 \Leftrightarrow 6.2y = 9.9 - 9.2 \Leftrightarrow y = \dfrac{7}{62} \approx \dfrac{1}{9} = 0.\overline{1}$ and since y is decreasing with x, $x > 4$ if $y < 0.1$.

6. A line passes through the points $(2, 2)$ and $(4, 1)$.

   (a) (6 points) Find the slope of the line.
   Solution: $m = \frac{\Delta y}{\Delta x} = \frac{y-2-y-1}{x_2-x_1} = \frac{1-2}{4-2} = -\frac{1}{2}$.

   (b) (6 points) Use the point slope formula to write an equation for the line.
   Solution: The formula is $y - y_1 = m(x - x_1)$. Plugging in, we have $y - 1 = -\frac{1}{2}(x - 4)$.

   (c) (6 points) Write the slope-intercept form ($y = mx + b$) for the equation of the line.
   Solution: Solving for y yields $y = -\frac{1}{2}x + 3$